CARTOGRAPHIC STUDY OF SPATIAL STRUCTURE OF PHENOMENA USING FRACTAL GEOMETRY

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ABSTRACT

In this paper there are presented two methods of spatial structure of phenomena. Both methods in their essential fundaments take into consideration internal cohesion of the phenomenon under studies.

Indicating fractal dimension using curdling method, that is proposed, enables to evaluate the compact's index of basic units in which phenomenon occurs. The higher concentration of units counted in succeeding stages the higher the value of fractal dimension. There were proposed a research in subareas and presentation of results in form of isoline map to evaluate the local character of spatial structure.

In the second method spatial arrangement of a phenomenon is evaluated on the basis of occurrence frequency of modules with fixed type of basic units in which phenomenon occurs. Patterns have been established regarding compact's index in 9-unit module. Maps of types complement information about compactness of phenomenon, which was acquired in the first method, with a data which types of units generate this cohesion.

INTRODUCTION

The separated areas of phenomena's distribution in the research area shown in the map, aerial or satellite photography create specific spatial arrangements.
Recognition of the character of objects and phenomena's distribution is usually limited to the study of diversity of their density and to presenting the results in the form of choropleth map.

For the transformations connected with the change of soil usage, the analysis and estimation of phenomenon's distribution in a research area are of great importance. A map, as a visual model of spatial relationships, is significant for the research. Cartographic presentation of surface objects is the most adjusted, for human perception, way of recognizing the structures and their distribution in the geographical space. For that purpose, such cartographical methods as isolines, choropleth or dasymetric are used (Klimczak 2000, 2001).

Broadening the structural realization in the form of cartographic models (the effect of different source data's processes) makes the recognition and evaluation of spatial structure of phenomena easier. Such an evaluation is a crucial issue while considering spatial management.

The evaluation of distribution, using statistic methods, theory of information, picture analysis in comparison with patterns and fractal geometry, is a great implementation of visual analysis of objects' allocation.
In the paper, the evaluation of surface objects' allocation using the fractal – box method, has been proposed. For estimating fractal dimension, the author's adaptation of curling method by Mandelbrot has been used. Index, estimated in such a way, characterizes cohesion of researched phenomena. Presented solutions take into account spatial aspect of phenomena distribution, due to indicating and comparing their local box dimension in adequately created sub areas.

What is also proposed in the paper, is broadening of the research by establishing typical patterns of neighbouring sub areas in 9–units modules using compact's index. Basic units, which create modules, have been classified taking into account 512 combinations of units in which the phenomena occurred. What is important are typical, characterizing spatial distribution, the most often met elementary patterns in neighbouring sub areas.
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CHARACTERISTICS OF RESEARCH AREA

Proposed methods of research have been used while evaluating spatial structure of the chosen forms of land cover in the region situated in the southwest part of Poland - Lower Silesia. In the south it borders on Czech republic and in the east, on Germany. The area has 19 948 km^2 and it includes two big geographical areas: Sudety Mountains with Sudety Foreland and Central Poland lowland (Silesia Lowland).

Lower Silesia region is typically agricultural. Agricultural area (the object of research) in the region is 1,1 mln ha. Arable lands are the biggest part of agricultural terrains (78%). The value of production area of lower Silesia is differentiated but still it is higher than Polish average. South part of the region makes the exception. It is the region with mountainous areas or hills, which makes agriculture difficult there. East part of the region is of low agricultural value as well, while in the north and central part there are regions of the highest agricultural value.

GIS (for accepted units), elaborated for the arable fields and green areas, has been used for evaluating structure of their distribution, using fractal geometry and phenomenon classification by adjusting patterns.
The forms of land cover, elaborated by Institute of Geodesy and Cartography and using satellite photos (in the frame of CORINE Land Cover programme), were the source materials for creating the database.

INDICATING FRACTAL DIMENSION IN THE SUB AREAS USING CURDLING METHOD

Fractal geometry is more and more popular when it comes to proposing new methods of analysis of environmental data. (Poetigen 1995). For complex arrangements (as the ones created by arable areas), solutions dealing with fractal geometry can be proposed while analysing their geometric features and distribution structure.
In the paper, the curdling method has been proposed for the analysis of surface objects distribution. This method is and author's adaptation of the method used while estimating the point elements' distribution (Klimczak 2003a, 2003b).

„Curdling" is the name, which Mandelbrot proposed for the process of fractal dust creation (Mandelbrot 1982). Fractal dust is a disconnected set of points, which have the feature of a cluster. The aim of the method is to evaluate the concentration level – density of the points inside the area of chosen units, which are created in the subsequent stages - the levels of narrowing of the initial unit in order to adjust to a set of other units, in which the concentration takes place. The initial area is narrowed by the subsequent divisions of units' network, in which there are points' clusters. Mandelbrot proposes curdling method as the method of process modelling, which makes galaxies and stars' clusters in the sky. He points that the stars' dimension is 1,23. He indicates as well the fact that if galaxies and stars were not fractal dust and if they were not equally distributed in space, then the sky at night would shine an even light. There would not exist any constellations people made out (Mandelbrot 1982).

Presented algorithm of obtaining data for indicating fractal dimension (Figure 1) can be described as follows. In the first stage, the network of initial units (it is the network of the longest side) is put on the researched area. The initial unit is divided into 9 units of the sides of 1/3 of the network side's length. What is left are the units in which there is at least one element (appoint, object, signature, pixel). We count those units. In the next stage, those units are divided into the next 9 parts (the side is of 1/9 of the basic network's length) and once again we count the units in which there is at least one element. The next stage of the division (the third one) is the division into the next 9 units and a choice of units, in which there is at least one element, narrows the localization to the area made of elements, which create a cluster of discreetly appearing elements – objects, which are situated in the unit of 792 parts of initial network. Those elements can be the neighbouring ones or they can be isolated. It means that the size of initial unit should be chosen in such a way as to make a multiple (792 times) of a studied element's size (point, object).

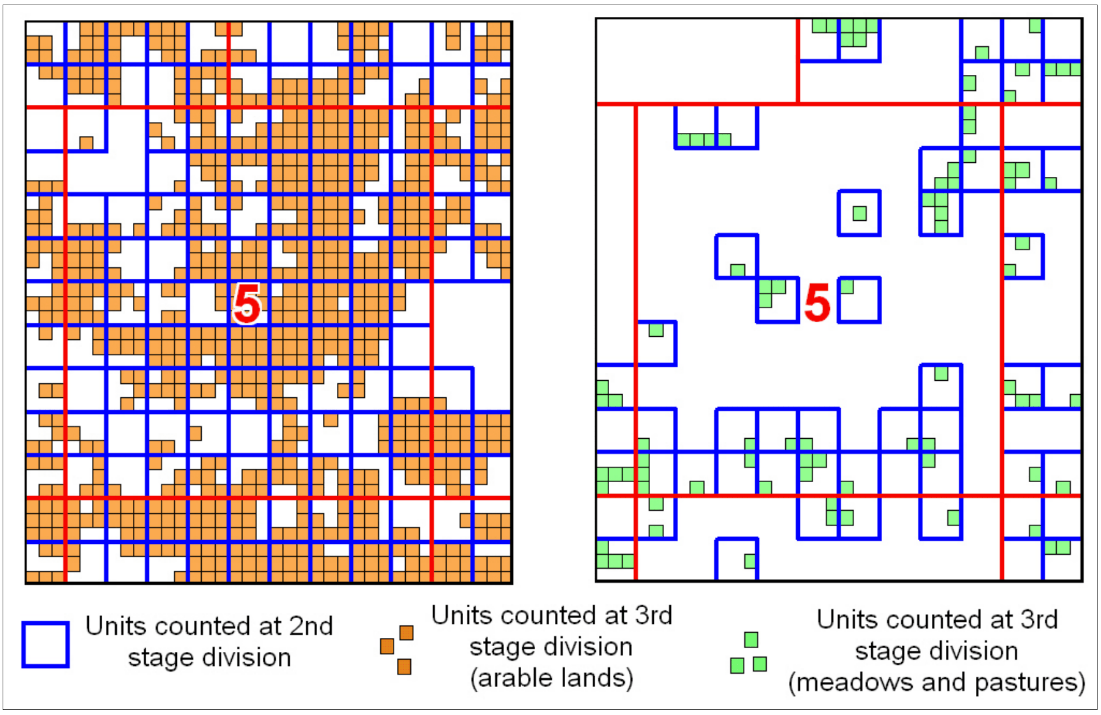

Figure 1: Unit division and counting in the curdling method, stage 2 and 3

Fractal dimension D_c evaluates the concentration level – cluster of elements treated as a fractal dust, and is calculated according to general formula for the box dimension. Because in the studies, the networks' sequence, in which the size of sides decreases thrice while transferring from one network to the other, is accepted, then, for the presented above stages of obtaining data, the calculations are made on the basis of the following formula (Bovill 1996, Klimczak 2003):

$$D_c\left[\text{box},(1/3)\text{-}(1/9)\right] = \frac{\left[\log(N_{1/9}) - \log(N_{1/3})\right]}{\left[\log 9 - \log 3\right]} \tag{1}$$

(for the first and second stage),

$$D_c\left[\text{box},(1/9)\text{-}(1/27)\right] = \frac{\left[\log(N_{1/27}) - \log(N_{1/9})\right]}{\left[\log 27 - \log 9\right]} \tag{2}$$

(for the second and third stage),

where: D_c – box dimension for the previous and next division stage,
$N_{1/3}$, $N_{1/9}$, $N_{1/27}$ – number of units left after each of the following stages of division.

In the proposed way of division, each network's unit is also divided into 9 smaller units, which sides' length equals 1/3 of the previous unit's side. When we use such a kind of network while calculating boxes – units, we obtain a sequence of numbers - $N(3^{-(k+1)})$, k = 1, 2, 3,… Box dimension, which is calculated using the inclination of the straight, which joins one set of data with the other one in the logarithm graph is as follows:

$$D_c = \log_3 \left[N(3^{-(k+1)}) / N(3^{-k})\right] \tag{3}$$

Calculated box dimension, in which the second and the third stages of division (box 1/9 and1/27) are taken into account, will be more precise than the one, which will be obtained, in the first and second stages (box 1/3 and 1/9).
The box dimension, calculated using the above formula, allows to estimate the process of concentration of studied elements.
The above mentioned formula for the second and third stage of the division can be presented as follows:

$$D_c = \log_3 (N_{1/27} / N_{1/9}) \tag{4}$$

Where – $(N_{1/27} / N_{1/9})$ stands for the average density of elements, treated as a fractal dust.

In the accepted system of division of the basic network, the value of the factor $[N(3^{-(k+1)}) / N(3^{-k})]$ in the range from 1 to 9 can be obtained. At the same time it is the number of possible elements in the unit from the next division into one unit from the previous division. Box dimension calculated for the integer values of the factor will give respectively:

$D_{C(1)} = 0$; $D_{C(2)} = 0{,}631$; $D_{C(3)} = 1$; $D_{C(4)} = 1{,}263$; $D_{C(5)} = 1{,}465$; $D_{C(6)} = 1{,}631$; $D_{C(7)} = 1{,}1{,}771$; $D_{C(8)} = 1{,}893$; $D_{C(9)} = 2$.

The value of the box dimension grows rapidly for the factor's value of 1-3. The same scope for the range 3-9 of the factor (resulting from the feature of the logarithm function) shows, that the box dimension is not well proportioned to objects' density. However, it allows to distinguish the specific character of the clusters of differentiated, coincidental level of concentration

INDICATING BOX DIMENSION FOR THE ARABLE LANDS AS WELL AS FOR THE MEADOWS AND PASTURES IN LOWER SILESIA

The proposed method has been used in practice while analysing the structure of arable lands, meadows and pastures in Lower Silesia. The box dimension, which enables the evaluation of concentration of those lands, has been calculated locally for the previously adjusted to research method sub areas. Studied lands are surface objects, so it was accepted to think of the areas, considered using reference unit (basic unit) of about 1 km x 1 km, as a fractal dust. Those units create a network elaborated for Poland according to Temkart system. They make reference units (graphic database) for the database, elaborated by the System of Geographical Information, which was used for automation of analyses' process. The research has been carried out accepting indication of fractal dust on the level of ≤ 50%. of the unit's surface. For those assumptions, the binary discretization of the vector picture of distribution has been made, using the basic units' network.

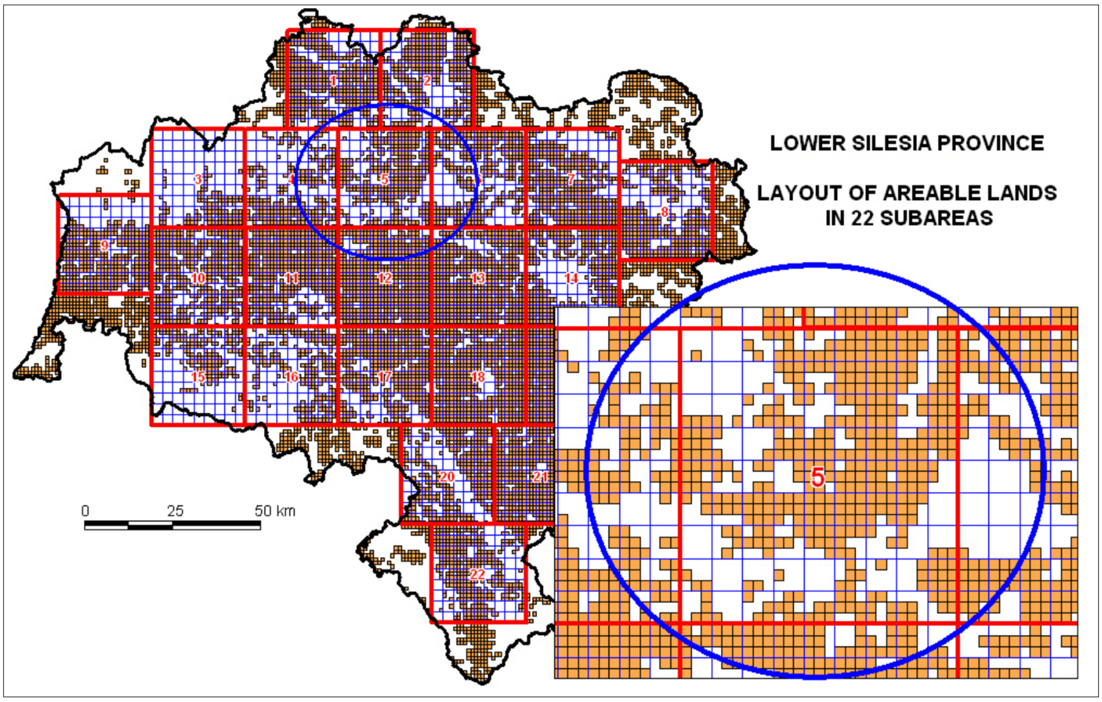

Figure 2: The distribution of 22 sub areas against the arable areas background (basic units 1x1 km) in Lower Silesia

The research has been carried out, in carefully chosen for both methods, sub areas. In the studied area, 22 sub areas were indicated. Their distribution is shown in fig. 2. Each sub area includes 729 basic units. They create the network of initial units. The units (sub areas), indicated in the first stage of division, have 81 km^2 each, in the second stage – about 9 km^2 and in the third one (basic units) – 1 km$^{2.}$. For the next divisions, in each sub area, the calculations were made and the box dimension was calculated. Obtained results are shown in the table 1.

Table 1: The box dimension calculated using curdling method and the coefficient of density for arable lands (AL) and meadows and pastures (M, P)

Number of subarea	N(1/27) for 50% (AL)	N(1/9) for 50% (AL)	D_c for 50% (AL)	Density coefficient (AL)	N(1/27) for 50% (M, P)	N(1/9) for 50% (M, P)	D_c for 50% (M, P)	Density coefficient (M, P)
1	2	3	4	5	6	7	8	9
1	439	72	1.646	60.2	35	20	0.509	4.8
2	402	70	1.591	55.1	64	32	0.631	8.8
3	146	35	1.300	20.0	20	9	0.727	2.7
4	307	65	1.413	42.1	72	33	0.710	9.9
5	400	74	1.536	54.9	39	25	0.405	5.4
6	377	74	1.482	51.7	54	28	0.598	7.4
7	455	69	1.717	62.4	35	23	0.382	4.8
8	417	74	1.574	57.2	17	10	0.483	2.3
9	399	65	1.652	54.7	13	10	0.239	1.8
10	497	81	1.651	68.2	23	16	0.330	3.2
11	550	80	1.755	75.4	20	14	0.325	2.7
12	644	81	1.887	88.3	12	10	9.166	1.6
13	582	81	1.795	79.8	21	16	0.248	2.9
14	430	71	1.639	60.0	108	33	1.079	14.8
15	186	44	1.312	25.5	84	35	0.797	11.5
16	269	68	1.252	36.9	123	54	0.749	16.9
17	495	78	1.682	67.9	20	12	0.465	2.7
18	604	80	1.840	82.8	9	8	0.107	1.2
19	694	81	1.955	95.2	15	10	0.369	2.1
20	386	73	1.516	52.9	23	15	0.389	3.2
21	582	79	1.818	79.8	5	5	0.000	0.7
22	382	69	1.558	52.4	41	28	0.347	5.6

The results of calculations have been presented also in the form of isoline map in Figure 3. It makes an illustration of distribution of studied phenomena, taking into account their inner cohesion, which is very important for spatial planning.

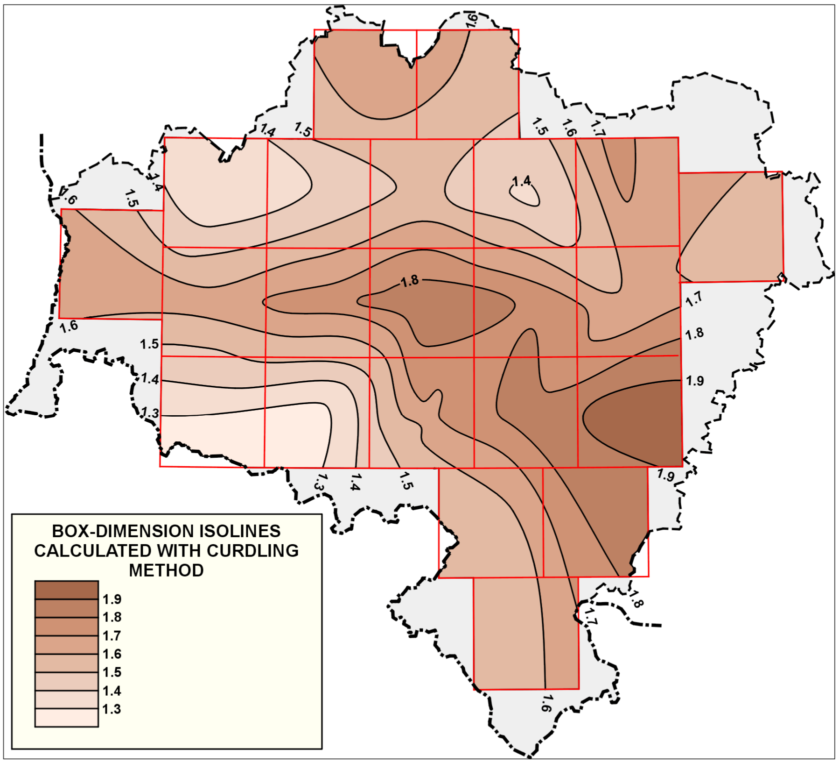

Figure 3: Isolines of box dimension calculated using curdling method for arable lands in 22 sub areas

Counted in 22 sub areas index for fractal dimension for arable lands, has the values from 1,252 to 1,955. 12 sub areas, situated mainly in the central and east part of studied area (as can be seen in pic. 3), are characterized by the high level of inner concentration of the phenomenon. It is the effect of vast amount of arable lands in that area. The average level of phenomenon’s intensity is from 6 to 9 units in 9-units module.

Different results and phenomenon’s distribution have been obtained for the group of green areas, such as meadows and pastures. The index of fractal dimension has the value from 0 to 1,079 there. Such a low level of concentration shows the low share of that group in the structure of land usage and of its great spread. The sub areas of the higher concentration level are situated in the south and north parts of the studied area.

PICTURE ANALYSIS

The studies of the spatial distribution of arable lands can be carried out, using their picture forms obtained using satellite or aerial techniques.
When it comes to cartographic form of presenting arable lands, differently situated areas make the layout. The recognition of the character of phenomenon’s distribution cannot be limited just to the studies of density or the size of the area in which there is a feature interesting for the researcher. What is important are the typical, characterizing the spatial distribution, repeated elementary layouts of the neighboring areas.

Adjusting to the pattern is the simplest way of classifying and recognizing the pictures. In order to use that method, special pattern for each picture which is to be recognized, is essential.

By placing the network of basic units on the analyzed area, and describing the units in which the phenomenon appears as 1, and the rest of the units as 0, we obtain complex picture of repeating patterns, encompassing the whole, considered area. The complex picture is divided into simpler sub-pictures – modules of constant size, which are recognized by comparing to the defined *a prori* patterns.

The studies of surface objects distribution can be carried out, analyzing the complex modules, for example, from 9 (3x3) elementary units of the area, which depends on the accepted level of simplification of natural borders of the phenomenon, to artificial borders of elementary units. The arrangements of the types of pictures, with he consideration of index of inner cohesion of the units in which the phenomenon appears, are presented in the works of the author (Klimczak 2003).

Proposed classification of the layouts has been used for analyzing the distribution of arable lands, meadows and pastures, in the studied area in Lower Silesia. Elementary units, which are the basis of discretization of appearing in clusters areas, have been accepted according to research (using fractal geometry) as 1 km x 1 km. The qualification of the feature in the reference unit has been established on the level of area’s share– ≥ 50%. 9-units modules create units of the sides of 3 km x 3 km. Studied area includes 1782 modules.

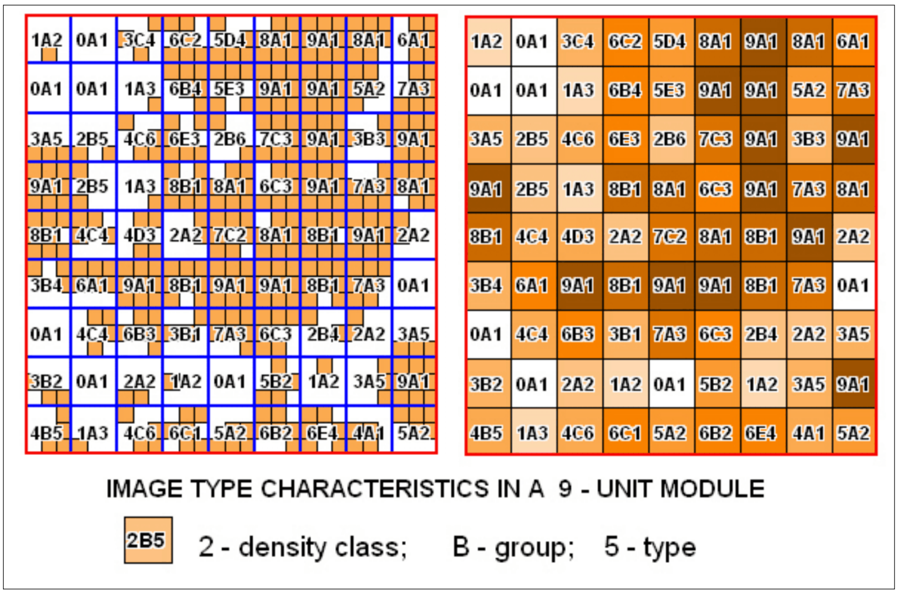

Figure 4: The types of layouts of arable lands in the subarea 5

In the table 2, the frequency of the most popular types in the whole researched area, have been compared. The table includes the breakdown of the types of arable lands and green areas. The kind of types for both areas is completely different. In arable fields there is a dominance of types from the group of high density (the first number of type), while in meadows and pastures – the types from the group of low density in module. In the researched region, in both areas, there are many modules without the phenomenon occurrence (types indicated as 0A1)

The presented way of analyzing the phenomena's distribution can be treated as implementation of curling method. The analysis of the picture, according to the accepted patterns, specify calculated fractal dimension by providing the information, which types decide of such a level of cohesion or spread of studied phenomena.

Table 2: Frequency of the most popular types of pictures of arable lands, meadows and pastures

Module type	Arable land		Module type	Meadows	
	Number	Frequency		Number	Frequency
1	2	3	4	5	6
9A1	418	0,235	1A3	135	0,076
8A1	142	0,080	1A2	103	0,058
7A3	97	0,054	2A2	39	0,022
8B1	76	0,043	2A1	14	0,008
1A3	68	0,038	1A1	12	0,007
2A2	60	0,034	3A5	11	0,006
6A1	50	0,028	2B1	11	0,006
1A2	39	0,022	2B2	10	0,006
5A2	37	0,021	3A3	9	0,006
4B5	37	0,021	3A4	8	0,004
0A1	218	0,122	0A1	1336	0,750

SUMMARY

The proposed methods of analyzing the surface phenomena distribution in sub areas enable to evaluate the structure of the studies phenomenon on different level of accuracy. Calculated, with the curling method, box dimension enables the evaluation of the cohesion but do not give any bases for distinguishing inner configuration of the layouts. In many works connected with the transitions of environment, the recognition of the spatial distribution structure of the set of surface elements in the studied area, is crucial. The analysis of the picture, by comparing it to the patterns, is a good solution, proposed in the paper. It allows to distinguish the local types of layouts, which are characterized by the biggest number of occurrences. The results of the studies of spatial distribution of phenomena, using curling method, can be presented in the form of the models made with the usage of the cartograms or izolines. The models in the form of the map of types can be created on the multi-variant level (by evaluating the density and cohesion of the classified layouts) depending on the need. They provide more detailed information about the structure of the phenomenon. They enable carrying out multi –variant analyses. They are extremely useful while evaluating the state of the environment, while analyzing the changes in the environment and while establishing the conditions of spatial balance of its components.

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Biographical sketch of Prof. Halina Klimczak

Prof. Halina Klimczak is a graduate of Geodesy Course at Agricultural University of Wrocław. In 1981 she got PhD in Technical Science. She is academic worker in the Department of Geodesy and Photogrammetry at Agricultural University of Wrocław, Poland.

In her research work she deals with usage of cartographic modelling in exploring natural environment; constructing GIS for land management. She was head and realized five projects sponsored by State Committe for Scientific Research. She is an author of 50 published works and many thematic maps.

Prof. Halina Klimczak in her academic work conducts courses in fields of geodesy, mathematical and thematic carthography and carthographic modelling. In 2003 she published a work entitled “Carthographic modelling in studies on spatial phenomenon layout”.

She works in ICA Commision on Gender in Carthography in which she is a representative of Poland.

She is married and has two sons. Her husband is academic worker and is a geodesist her older son academic worker at University of Economics in Wroclaw and her younger son is studing at this university. Her hobby is tourism and literature.

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