Multi-scale Automatic Extraction of Terrain Structure Lines Based on Wavelet Analysis

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Abstract

Terrain structure lines (valleys and ridges) are very important for geomorphological characterization, they construct the skeleton of terrain undation and variation, and they are scale-dependent because the ones are more important at a larger geographical scale than their counterparts at a smaller. Wavelet analysis is a new branch of mathematics burgeoning at the end of 80s. It has double meanings simultaneously on profundity of theory and extent of application. Because it has good local character at both time or space and frequency field simultaneously, and sample interval of signal can be adjusted automatically with different frequency components, wavelet analysis method can adapt to different type of relief at the same time, such as mountainous or plain terrain or the both. This paper formulates briefly the basic principle of multiresolution analysis(MRA) on wavelet transform, represents a new method of multi-scale automatic extraction of terrain structure lines from contour data. It employs curvature to describe contour data through curve spline fitting, then decomposes curvature based on the wavelet MRA at multiple scales, thereafter determines the contributive size of curvature by inspecting the wavelet coefficients at different scale and records their corresponding positions exactly at which describes the degree of convexity and concavity. Accordingly, we can get the candidate point sets belong to the structure lines at different scale. Assuming the higher elevation is to the left and the lower is to the right along a contour line, the positive curvature represents the convexity, namely ridges, the negative represents the concavity, namely valleys. Therefore, the point sets belong to valleys or ridges can be sorted out easily from the candidate point sets at different scale. According to the basic rules of relief contour representation, multi-scale valleys and ridges can be determined from the point sets belong to valleys or ridges by identifying ascription of bisector at concave corner or convex corner on adjacent contours. Some practical examples are given to exam the method. Finally, this paper also discusses the efficiency and accuracy of the algorithms, and simply compares it with existing method.

1. Introduction

The wide availability of topographic data in digital form has increased interest in terrain modeling and has caused a rapid evolution of Geographic Information System (GIS). Terrain structure lines (valleys and ridges) are very important for geomorphological characterization, they construct the skeleton of terrain undation and variation. How to automatically extract the

structure lines from digital terrain data is one of the most important issues when working with terrain models and extending the ability of GIS.

The methods of extracting the structure lines vary according to the model such as regular square grids (RSGs) and triangular irregular networks (TINs) and contour line model (CLM). Furthermore they are scale-dependent (Wood 1999) because the structure lines are more important at a larger geographical scale than their counterparts at a smaller (Gallant and Hutchison 1997). For example, the terrain structural characteristics emerge obviously when at a larger geographical scale, and they generally represent the patterns of tree branches.

Wavelet analysis is a new branch of mathematics burgeoning at the end of 80s. It has double meanings simultaneously on profundity of theory and extent of application. Because it has good local character at both time or space and frequency field simultaneously, and sample interval of signal can be adjusted automatically with different frequency components, wavelet analysis method can adapt to different type of relief at the same time, such as mountainous or plain terrain or the both, and its anti-noise ability is strong. This paper represents a new method of multi-scale automatic extraction of terrain structure lines from contour data based on wavelet MRA .

2. Formulation of the basic principle of multiresolution analysis(MRA) on wavelet transform

A set of close subspaces $\left\{V_j\right\}_{j\in Z}$ in space $L^2(R)$ are defined as a multiresolution analysis(MRA) in $L^2(R)$ when：

①monotonousness: $V_j \subset V_{j+1}, \forall j \in Z$;

②approach: $\cap V_j = \left\{0\right\}, \overline{\cup V}_j = L^2(R)$, $j \in Z$;

③scaling: $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}, \forall j \in Z$;

④translation invariance $f(x) \in V_0 \Rightarrow f(x-k) \in V_0, \forall k \in Z$;

⑤existing of Riesz basis: $\forall g \in V_0$, making $\left\{g(x-k) \,|\, k \in Z\right\}$ to construct the Riesz basis in V_0.

Assuming $\left\{V_j\right\}$ as a MRA, φ and ψ are the relative scale and wavelet function respectively, $h(n)$ and $g(n)$ represent their orthogonal filter groups, they have lower and high pass characters. Defining the close subspace W_j as the complement of V_j in V_{j+1}, then the decomposition and reconstruction of 1-D Mallat algorithm are:

$$V_{j+1} = V_j \oplus W_j$$

where $V_j = \overset{j-1}{\underset{l=-\infty}{\oplus}} W_j$ and $L^2(R) = \overset{+\infty}{\underset{l=-\infty}{\oplus}} W_j$.

If $f(x) \in L^2(R)$ then $f_{J-1} \in V_{J-1}$ and $g_{J-1} \in W_{J-1}$. Defining A_j, D_j as the projection

arithmetic operators from $L^2(R)$ to the subspaces V_{j+1}, then

$$f = f_J(t) = f_{J-1}(t) + g_{J-1}(t) = A_0^d f = A_{-J}^d f + \sum_{-1}^{-J} D_j f$$

where its approximation $A_j^d f$ at scale j (or 2^j):

$$A_j^d f =< f(x), \varphi_{j,k}(x) >= \sum_k h(k - 2n) A_{j+1}^d f$$

and its detail $D_j f$ at scale j (or 2^j):

$$D_j f =< f(x), \psi_{j,k}(x) >= \sum_k g(k - 2n) A_{j+1}^d f$$

namely $f(x)$ can be decomposed into { $A_{-J}^d f, (D_j f)_{-J \le j \le -1}$ }.

$A_{j+1}^d f$ can be reconstructed from $A_j^d f$ and $D_j f$:

$$A_{j+1}^d f = \sum_k h(n - 2k) A_j^d f + \sum_k g(n - 2k) D_j f$$

Therefore data sets to be analyzed can be decomposed into two sections at every scale, namely approximations $A_j^d f$ and details $D_j f$, in pyramid using Mallat algorithm. The approximations represent the gentle or smooth components of data; the details represent abrupt or local ones.
The procedure of detecting the abrupt positions of a sign is usually to execute wavelet MRA at first, the module of coefficients will be max after wavelet transformation when the sign changes suddenly, and thus the abrupt positions of the sign can be identified through the max module.

3. Automatic extraction of the candidate point sets belong to the structure lines

There are various of methods for automatic extraction of the structure lines based on CLM, such as identifying the contour curvature; tracing the contour perpendiculars; extracting the contour skeletons; extracting the contour skeletons based on voronoi diagram; and so on. Because most of them have lower anti-noise ability, it is difficult to extract the structure lines in the regions of fragment terrain or with no smooth contours. The main idea of multi-scale method when extracting the structure lines from the terrain model is that identifying local structural point sets is at lower scale but global ones at higher scale. Therefore the method has

a higher anti-noise ability.

3.1 Curvature computation of the contour data and its decomposition based on the wavelet MRA at multiple scales to identify the global and local point sets

Curvature is used to represent linear features in many domains. The bend characteristics and the orientation variant of curve can be described clearly with curvature. Furthermore it has invariant property in rotation and shift, so it is suitable to be analyzed by using wavelet.

Curvature in a contour line cannot be defined at every position because the contour line is composed of discrete points and not enough to smooth. Therefore, the curvature can be computed through the curve fitted by using the segment cubic spline or quadratic function or through the approximated method (Wu 1999).

3.1.1 Calculate curvature by using the segment cubic spline fitting

Define $C = \{x(t), y(t)\}$ as a curve where

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$$

According to $y' = \dfrac{dy}{dx}$, $y'' = \dfrac{d^2 y}{dx^2}$ and the curvature $k = \dfrac{y''}{(1 + (y')^2)^{3/2}}$

Define $\dot{x} = \dfrac{dx}{dt}$, $\ddot{x} = \dfrac{d^2 x}{dt^2}$, $\dot{y} = \dfrac{dy}{dt}$, $\ddot{y} = \dfrac{d^2 y}{dt^2}$

then the curvature $k = \dfrac{\ddot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

The parameter t can be segmented in discrete points of the data sets or in equivalent arc length along the curve.
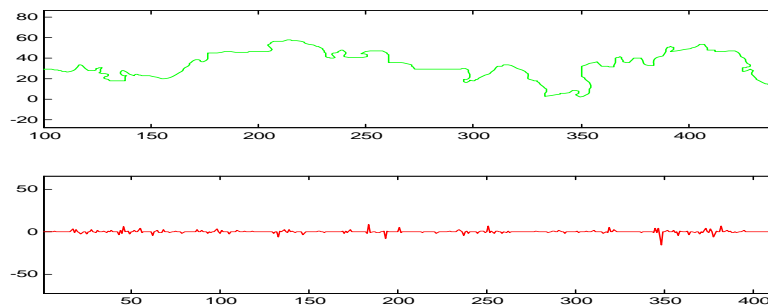

Figure 1 the curvature distributions of the test curve calculated with the cubic spline fitting

3.1.2 Decomposition of the curvature based on wavelet MRA at multiple scales and identification of the global and local point sets

The curvatures can thereafter be decomposed at multiple scales by using Mallat algorithm mentioned above so as to identify the global and local point sets along to the structure lines.

Figure 2 demonstrates the decomposition result of the curvatures. The size of the composition values, namely wavelet coefficients, and their distributed position are clearly represented. Now the contributive size of curvature can be determined by inspecting the module of wavelet coefficients at different scale and recorded their corresponding positions exactly at which describes the degree of convexity and concavity. Accordingly, we can get the candidate point sets belong to the structure lines at different scale. The larger module usually implicates the contour line is more curvilinear at its position, and the point sets at higher scale are generally more important than ones at lower scale. So the point sets at higher scale are called the global points and at lower scale the local points reversedly. The structure lines bounded by the global points generally consist of main pattern of the terrain model.
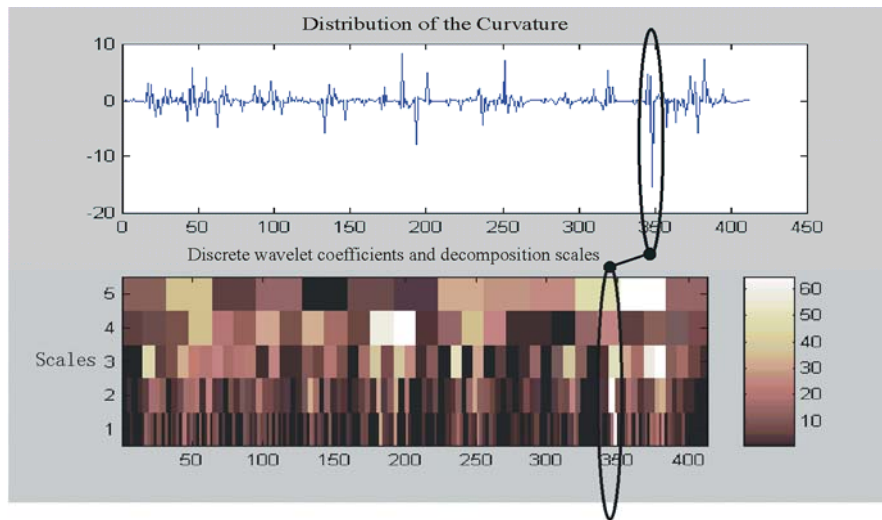

Figure 2 demonstrates the decomposition result of the curvatures based on wavelet MRA

3.2 Identification of the candidate point sets belong to the valleys or ridges

Classifying the candidate point sets belong to the valleys or ridges employs identification of the shape of bend. According to the formula of calculating curvature mentioned above, assuming the higher elevation is to the left and the lower is to the right along a contour line, the positive curvature represents the convexity, namely ridges, the negative represents the concavity, namely valleys. Therefore, the point sets belong to valleys or ridges can be sorted out easily from the candidate point sets at different scale.
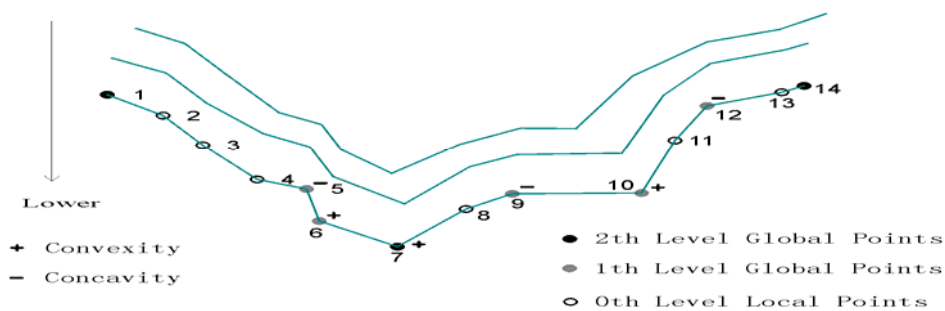

Figure 3 Identification of the candidate point sets belong to the valleys or ridges

4. Determination of the valleys and ridges

Determination of the valleys and ridges needs to link the candidate point sets belong to the valleys or ridges respectively. According to the basic rules of relief contour representation, multi-scale valleys and ridges can be determined by identifying ascription of bisector at concave corner or convex corner on adjacent contours. The basic principle of linking the valleys and ridges from the point sets as follows:

(1) The reference point and the point to be identified have the same convexity or concavity and they are the same level global points or local points;

(2) The distance S between the reference point and the point to be identified should be limited to a threshold;

(3) The line of linking the reference point and the point to be identified should locate the included angle α of the contour in the reference point;

(4) The patulous directions of the contour in the reference point and the point to be identified should be near. It can be implemented through identifying the difference β between the two bisectors ascribed to the adjacent contours in the reference point and the point to be identified respectively, namely whether the difference is smaller than the given threshold.

(5) The variation γ of the angle of the valleys or ridges in the point to be identified should be smaller than the given threshold. The variation represents the difference between the segments just linked and to be linked on the structure lines.

To identify the candidate point whether to be linked, we may use the assessment model compounded by multiple factors using fuzzy technology (Yang 1998):

$$U = a_1 r_{11} + a_2 r_{21} + a_3 r_{31} + a_4 r_{41} = \sum_{i=1}^{4} a_i r_{i1}$$

Where a_i are the weights of the identification function, and r_{i1} are the identification function $f(S, \alpha, \beta, \gamma)$ with factors described above.

To facilitate procedure of the linking, according to the scale the relevant global points at a higher level must be linked at first so as to shape the main structure lines, thereafter the relevant local points be linked or ignored as need.

The extraction of the valleys searches every contour from the high elevation to the low in terrain model. The basic procedures for linking valleys (the principle of linking ridges is the same) are as follows:

Step 1: Sort out the valley point not linked that has the highest elevation;

Step 2: Find a valley point on the contour whose elevation is lower than the point and their elevation difference is the smallest, and assess every point synthetically. Then identify the point as current point, which has the largest assessment value, and it belongs to the structure line.

Step 3: Link the current point to that one in step 1, they consist of a segment of the structure line;

Step 4: Begin at the current point, continue to search on the next contour, and iterate step 2 and

3;

Step 5: The linking search of this valley ends if the next point of the current point locates the other valley or cannot be picked out.

Step 6: iterate step 1 to 5 above, continue to search the next valley till end.

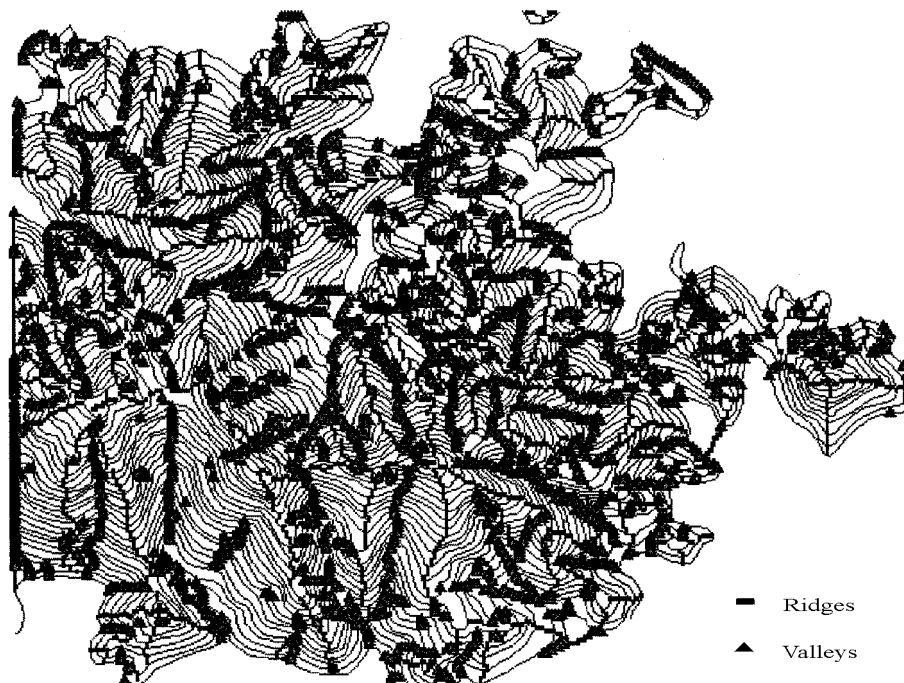The test example is demonstrated as figure 4.

Figure 4 the valleys and ridges extracted automatically from the test terrain data

5. Conclusions

This paper provides an approach for multi-scale automatic extraction of the structure lines from contour data based on wavelet transform. The main idea of multi-scale method is that identifying the global structural point sets is at higher scale but the local ones at lower scale. This facilitates the procedure of linking valleys and ridges and not need to eliminate the redundant points on smaller bend in advance. Therefore the method has a higher anti-noise ability and a broader availability. The edit tools need to be developed so as to link the shorter or discontinuous segments of the structure line in future work.

Acknowledgements

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