Transformations of Spatial Information in Multi-scale Representation

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Abstract

Map is media for recording geographical information. The information contents for a map is of interest to spatial information scientists. In this paper, existing quantitative measures for map information have been evaluated. It has been pointed out that these are only measures for statistical information and some sort of topological information. However, these measures have not taken into consideration of the spaces occupied by map symbols and spatial distribution of these symbols. As a result, a set of new quantitative measures is proposed, i.e. for metric information, topological information and thematic information. Experimental evaluation is also conducted. Results show that the metric information is more meaningful than statistical information and the new index for topological information is also more meaningful than the existing one. It is also found that the new measure for thematic information is also useful in practice. For the metric information on maps with different scales, the ratio of entropy value is more or less linearly proportional to the root-square of the scale ratio.

1. Introduction

For many centuries, map has been as a media for recording and presenting geographical information and has played an important role in human activities. On a map, geographical information is expressed with cartographic symbols. As map is regarded as a communication tool, cartographers are interested in the effectiveness of map design and the information content of a map (e.g. Knopfli 1983, Bjørke 1996). The former can be studied either through theoretical analysis or through map evaluation experiments similar to a clinic survey. However, it is outside of the scope of this study and no further discussion will be conducted in this paper. Indeed, this paper is to discuss the information content of a map.

Maps are usually associated with scale. In each country or region, maps are normally produced in a series, with different scales. The typical map scales are 1:1,000, 1:5,000, 1:10,000, 1:50,000, 1:100,000 and so on. The maps at smaller scales are usually derived from maps at a larger scale to avoid the duplication of the expensive field (new) survey. The process to derive maps at a smaller scale from maps at a larger scale is referred to as map generalisation. In this paper, attention will also be paid to the variation of information contents in maps with scales.

The interest in map information dated back to later 1960s after the publication of the work on quantitative measures of information by Shannon (1948) and Shannon and Weaver (1949), which is normally termed as “information theory” and was applied in communication theory. 'Entropy' is a quantitative measure for the information content contained in a message. The pioneering work in quantitative measurement of map information was done by Sukhov (1967, 1970), who considered the statistics of different types of symbols represented on a map. That's the entropy of these symbols are computed. This is a kind of direct application of Shannon's information measure in cartography. This is indeed a kind of statistical information. Later, Neumann (1987, 1994) did some work on topological information of maps. Neumann (1994) demonstrated the measurement of topological information for a contour map using the information concept developed in communication theory. In his work, a dual graph is formed to record topological relationship between neighbouring contour lines, and then the entropy of the dual graph is computed. Quantitative measures for map information has bee used for comparing the information contents between maps and images, maps at different scale, evaluation of map design and so on (Knopfli 1983, Bjørke 1996).

However, it is clear that spatial information is more than simple statistical information and topological information. It may also contain geometric and thematic information. In other words, the spatial position and distribution of map symbols should also be considered when a quantitative measure is designed for spatial information. In this study, Voronoi regions of symbols have been employed to model the spatial distribution of map symbol and then a new set of quantitative measures for spatial information on a map.

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This introduction is followed by an evaluation of existing measures (Section 2). Based on the evaluation results, a set of new quantitative measures is then introduced (Section 3) and these new measures are experimentally evaluated (Section 4). The change in information content of maps at various scale will be described in Section 5. Finally, some conclusions are made in Section 6.

2. Evaluation of existing quantitative measures for map information

As stated in the introduction section, two important pieces of work on map information have been done previously, one for statistical information and the other for topological information. In order to introduce new measures, it seems pertinent to conduct an evaluation on existing work to reveal the advantages and shortcomings of such measures.

2.1 The quantitative measure of information in communication: Entropy

Shannon (1948) was the first person to introduce entropy in the quantification of information. He employed the probabilistic concept in modelling message communication. He believed that a particular message is one element from a set of all possible messages. If the number of messages in this set is finite, then this number or any monotonic function of this number can be regarded as a measure of the information when one message is chosen from the set, all choices being equally likely. Based upon this assumption, information can be modelled as a probabilistic process. He then introduced the concept of 'entropy' to measure the information content.

Let X be the random message variable, the probabilities of different message choices are P_1, P_2, … P_i, … P_n. The entropy of X can be computed as follows:

$$H(X) = H(P_1,\ P_2, ...P_n) = -\sum_{i=1}^{n} P_i \ln(P_i) \qquad (1)$$

Statistically speaking, H(X) tells how much uncertainty the variable X has on average. When the value of X is certain, P_i=1, then H(X)=0. H(X) is at its maximum when all messages have equal probability.

In communication theory, three types of information are identified, i.e. syntactic, semantic and pragmatic information. Indeed, most of researchers in spatial information science try to follow these three types of information for a map.

2.2 Statistical information of a map: entropy of symbol types

Sukhov (1967, 1970) has adopted the entropy concept for cartographic communication. In such a work, only the number of each type of symbols represented on a map is taken into account. Let N be the total number of symbols on a map, M the number of symbol types and K_i is the number of symbols for i^{th} type. Then $N = K_1 + K_2 + ... + K_M$. The probability for each type of symbols on the map is then as follows:

$$P_i = \frac{K_i}{N} \qquad (2)$$

where, P_i is the probability for i^{th} symbol type, i =1, 2, …. M.

The entropy of the map can be calculated as follows:

$$H(X) = H(P_1, P_2, ..., P_M) = -\sum_{i=1}^{M} P_i \ln(P_i) \qquad (3)$$

The shortcomings of this measure for map information could be revealed by Figure 1, which is modified from (Knopfli 1983). Both map consist of three types of symbols, i.e. roads, buildings and trees and have exactly the same amount of symbols for each type. That is, there is a total of 40 symbols, i.e. 7 for roads, 17 for buildings and 16 for trees. Therefore, according to definitions in Equations (2) and (3), both maps shown in Figure 1 have the same amount of information, i.e. H=1.5. However, the reality is that the distributions of symbols on these two maps are very different. In Figure 1(a), the map symbols are mostly located on the right side of the diagonal along lower/left to upper/right direction and the tree symbols are scattered among buildings. Two rows of buildings are along the main road. However, in Figure 1(b), there is an area of trees on the left side of diagonal along the lower/left to upper/right direction and there is an area of buildings on the opposite direction. The roads are almost along the diagonal. Indeed, they represent different natures of spatial reality.

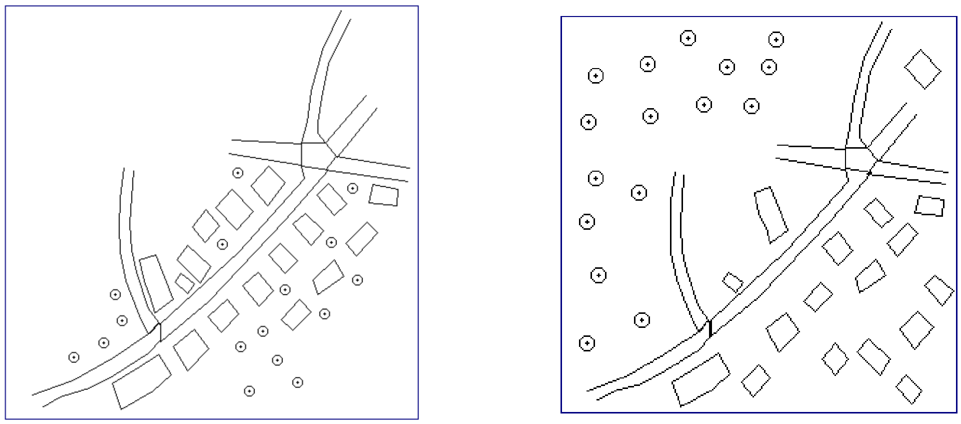


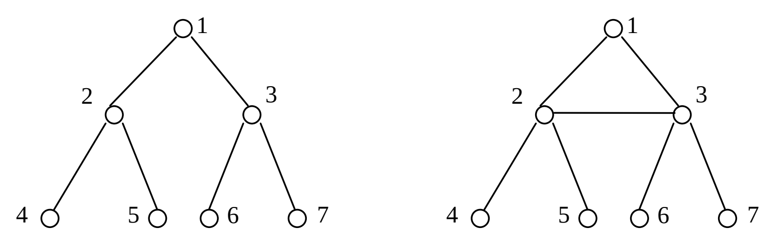
Figure 1. Two maps with the same amount of symbols but with different distribution

In other words, the entropy computed in this way only takes into account the number of symbols for each type but the spatial arrangement of these symbols is completely neglected. Such a value is purely statistical and thus is termed as "statistical information" in this paper. Indeed, it doesn't mean much in a spatial sense. Therefore, the usefulness of such a measure is doubtful.

2.3 Topological information of a map: entropy of neighbourhood

Neumann (1994) proposed a method to estimate the topological information of a map. The method consists of two steps: (a) to classify the vertices according to some rules, such as their neighbouring relation and so on, to form a dual graph, and (b) to compute the entropy with Equations (2) and (3). The method for the generation of such a dual graph was put forward by Rashevsky (1955).

Figure 2(a) shows a dual graph which consists of seven vertices at three levels. There are three types of vertices if classified by number of neighbours. There are four vertices with only one neighbour, one vertex with two neighbours and two vertices with three neighbours. Then, N=7, M=3, thus, the probabilities of these three types of vertices are: $\frac{4}{7}, \frac{1}{7}$ *and* $\frac{2}{7}$. The entropy of this dual graph is then computed using Equation (3) and the result is 1.38.



(a) A tree type of dual graph (b) A dual graphs with network

Figure 2 Dual graphs for computation of topological information

Let's now do a little change with this graph by connecting the two vertices in the second level, as shown in Figure 2(b). In this graph, there are four vertices with only one neighbour, one vertex with two neighbours and two vertices with four neighbours. The resultant entropy of this graph is exactly the same as that for Figure 2(a), i.e. 1.38. But, it is clear that the graph shown in Figure 2(b) is more complex than that in Figure 2(a). Thus, such topological information may not be able to reflect the true complexity of neighbour relations.

The question arising is "how to form a dual graph for a given map?" It is, indeed, a difficult task to produce such a dual graph, e.g. for the map given in Figure 1, because most of map features are disjoint. River network may be the type of feature convenient to form a dual graph. Indeed, in his study, Neumann (1994) produced a dual graph for river network. He also tried to produce a dual graph for contour lines. This is possibly because contour lines are nicely ordered according to their heights. Therefore, the usefulness of this method might be limited. Apart from this, the entropy computed by this method is only for the distribution of vertex type and has little relation with the topological relation.

2.4 Other types of information for a map

In fact, the usefulness of such a topological information has also been questioned by Bjørke (1996). He provides another definition of topological information by considering the topological arrangement of map symbols. Instead of one entropy name, he used a set. He also introduced some other concepts such as positional entropy and metrical entropy. "The metrical entropy of a map considers the variation of the distance between map entities. The distance is measured according to some metric" (Bjørke 1996). He also suggests to "simply calculate the Euclidean distance between neighbouring map symbols and apply the distance differences rather than the distance values themselves". "The positional entropy of a map considers all the occurrences of the map entities as unique events. In the special case that all the map events are equally probable, the entropy is defined as $H(X) = \ln(N)$, where N is the number of entities.

3. New quantitative measures for spatial information on map

In the previous section, existing measures for information contents on a map have been reviewed and evaluated. Their limitation should be clear. It is now pertinent to introduce new measures in this section, which should be sound in theory. The usefulness in practice will be evaluated in Section 4.

3.1 The line of thought

The communication theory is about human language, which don't have spatial component. Therefore, it could be dangerous to follow the line of thought developed in communication theory. That is, a completely new line of thought must be followed.

It is a commonplace that a map contains for following information about features:

(a) (Geo)metric information related to position, size and shape;
(b) thematic information related to the types and importance of features; and
(c) spatial relations between neighbouring features implied by distribution

Therefore, a set of measures needs to be developed, one for each of these three types, i.e. metric, topologic and thematic information.

To consider metric information, the position of a feature is not a problem. On the other hand, the consideration of size and shape is not an easy job. One approach to describe the size is simply based on the size of the symbol. However, serious deficiency with this absolute approach rest in its ignorance of the following facts:

(a) the size of a point symbol is always smaller than an areal symbol;
(b) the relative space of a feature, i.e. the empty space surrounding the feature, makes the feature separated from the rest. The larger the empty space surrounding the feature has, the more easily it can be recognised.

As map features share empty space surrounding them. It is necessary to determine the fair share for each feature. In this case, the map space needs to be tessellated by feature-based tessellation (Lee et. al., 2000). The Voronoi diagram seems to be the most appropriate solution. A Voronoi diagram is essentially a partition of the 2-D plane into N polygonal regions, each of which is associated with a given feature. The region associated with a feature is the locus of points closer to that feature than to any other given feature. Figure 3 shows the Voronoi diagram of the maps shown in Figure 1. The polygonal region associated with a feature is normally called the ‘Voronoi region’ (or Thiessen polygon) of that feature and it is formed by perpendicular bisectors of the edges of its surrounding triangles. Such a Voronoi region is a ‘region of influence’ or ‘spatial proximity’ for a map feature. All these Voronoi regions together will form a pattern of packed convex polygons covering the whole plane (neither any gap nor any overlap). Thus a Voronoi diagram of a map feature is its fair share of its surrounding space.

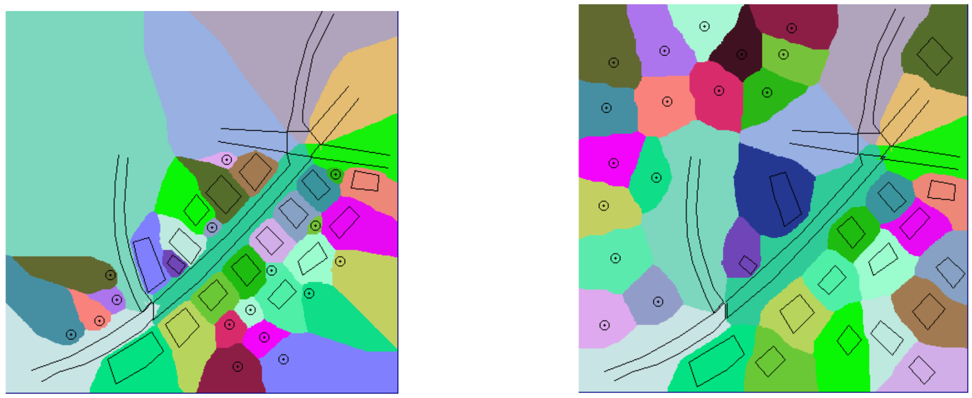


Figure 3 Voronoi diagrams of the maps shown in Figure 1

Indeed, Voronoi region is not only adequacy for the determination of the fair share of surrounding empty space for a map feature but also good for neighbour relationship (Gold, 1992). This is because the Voronoi region of a feature is determined by two factors, i.e. (a) the size of feature and (b) the neighbouring features. Indeed, Chen et al. (2001) have used Voronoi regions to describe spatial relations between map features.

For these reasons, the authors try to relate spatial information of map features to their Voronoi regions to develop a set of new quantitative measures. However, detailed discussion of the formation of Voronoi regions is outside the scope of this paper. Algorithms for generation of Voronoi region in vector mode have been presented by Okabe, et al. (1992) and a raster-based algorithm has recently been proposed by Li et al. (1999). Therefore, no further discussion on this topic will be presented in this paper.

3.2 Metric information on map: entropy of Voronoi regions

Metric information here considers the space occupied by map symbols only. In this case, an analogy to the entropy of binary image is used. That is, if the space occupies by each symbols is similar, the map has larger amount of information. If the variation is very large, the amount is smaller. This can be achieved by using the ratio between Voronoi-region of a map system over the enclosed area of the whole map as the probability used in the entropy definition. Let S be the whole area and and it is tessellated by S_i, i=1, 2, … N. Then, such a probability can be defined as follows:

$$P_i = \frac{S_i}{S} \quad (4)$$

The entropy of the metric information, donated as H(M), can then be defined as follows:

$$H(M) = H(P_1, P_2, ..., P_n) = -\sum_{i=1}^{n} \frac{S_i}{S}\left(\ln S_i - \ln S\right) \quad (5)$$

H(M) has its maximum when P_i has the same value for all i=1, 2, …N. In other word, when the area S_i is equal. Mathematically,

$$H(M)_{\max} = H(P_1, P_2, ..., P_n \,|_{P_1=P_2=...=P_n}) = \log_2 n \quad (6)$$

For example, the two maps shown in Figure 4 have different amount of metric information although both are tessellated by 9 polygons. The map in Figure 4(b) has the maximum H(M) for any tessellation into 9 polygons.

In the case of map, it is clear that for the same number of feature, the entropy will be larger is the symbols are more evenly distributed. However, it is clear that such entropy is related to the number of map symbols and thus it would be not convenient to compare two maps with different number of symbols. In order to overcome this shortcoming, the entropy could be normalised as follows:

$$H(M)_N = \frac{H(M)}{H(M)_{\max}} \quad (7)$$

Another possible measure is the ratio, R_M, between mean of the areas (m_A) and the standard deviation σ_A.

$$m_A = \frac{1}{n}\sum_{i=1}^{n} A_i \tag{8}$$

$$\sigma_A = \sqrt{\frac{\sum_{i=1}^{n}\left(A_i - m_A\right)^2}{n-1}} \tag{9}$$

$$R_M = \frac{m_A}{\sigma_A} \tag{10}$$

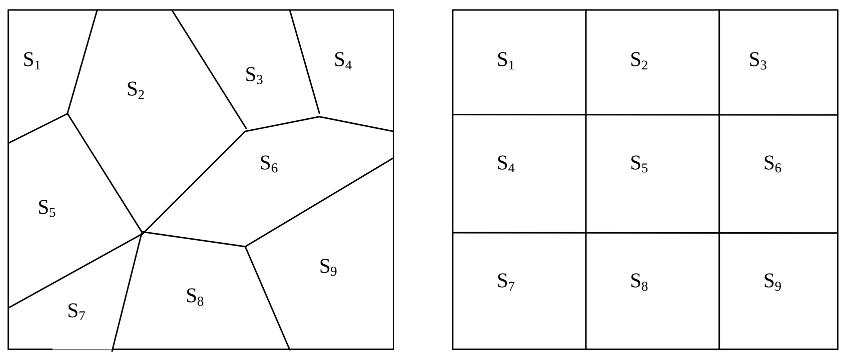


Figure 4 Two different tessellation of an area, resulting in two different amount of metric information

3.3 Topological information on map: Voronoi neighbours

As has been discussed in the previous section, the construction of dual graph for map features is a difficult task because the vast majority of map features are disjoint. However, with the Vonoroi region, all features have been connected together to form a tessellation. The generation of dual graph for map features could be replaced by the dual graph of the Voronoi region of these features. This is illustrated in Figure 5. Figure 5(a) is the Voronoi region and Figure 5(b) is the corresponding dual graph.

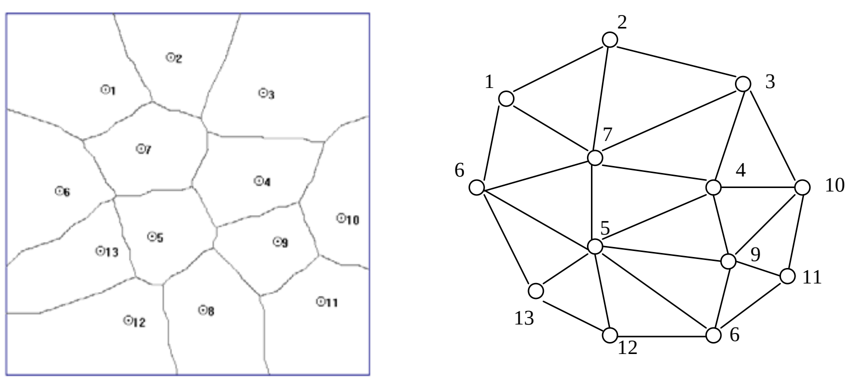


Figure 5 A Voronoi diagram and its dual graph

Then, the entropy of this map can also be computed as did for the graphs in Figure 2.

It has also been discussed before the entropy computed using the number of nodes in the graph is that of the distribution of different kind of vertices. It does not really reflect the complexity of dual graph directly. Indeed, sometimes, it is misleading, as shown in the case of Figure 2. Therefore, a new index needs to be designed. As the complexity of a dual graph can be indicated by the number of neighbours for each vertex, therefore, this number is a good measure already. In order to compare the complexity of dual graph with different vertices, the average number of neighbours for each vertex may be used as a value to indicate the complexity of a dual graph.

Let, N_s be the sum of the numbers of neighbours for all vertices and N_T the total number of vertices in a dual graph. Then, the average number of neighbour for each vertex is:

$$H_T = \frac{N_s}{N_T} \qquad (11)$$

3.4 Thematic information on map: Entropy of neighbour types

Thematic information related to the thematic types of features. It is understandable that, if a symbol has all neighbours with the same thematic hype, then the importance of this symbol is very low, in terms of thematic meaning. In the other hand, if a symbol has neighbours with different thematic types, it should be regarded as having higher thematic information. [Here, the neighbours are also defined by the immediately-neighbouring Voronoi regions]-. Based on this assumption, the thematic information of a map symbol can then be defined. Suppose, for the i^{th} map symbol, there are in total N_i neighbours and M_i types of thematic neighbours. There are in total n_j neighbours for j^{th} thematic type. Then the probability of the neighbours with j^{th} thematic type is as follows:

$$P_j = \frac{n_j}{N_i}, \qquad j = 1, 2, \ldots M_i \qquad (12)$$

The thematic information of the ith map symbol is then as follows:

$$H_i(TM) = H(P_1, P_2, ..., P_{M_i}) = -\sum_{j=1}^{M_i} \frac{n_j}{N_j} \ln\left(\frac{n_j}{N_j}\right) \qquad (13)$$

Suppose there are in total N symbols on a map, the total amount of thematic information for this map is then

$$H(TM) = \sum_{i=1}^{N} H_i(TM) \qquad (14)$$

4. A comparative analysis through experiments

In the previous section, a set of new measures has been proposed for the spatial information of a map. It is appropriate to conduct some experimental tests on the usefulness of these new measures and also to see whether these new measures are more meaningful than existing ones.

4.1 Metric information vs statistical information

The first test is on metric information. The two maps in Figure 1 were used. The corresponding Voronoi regions are shown in Figure 3. The results for the entropy of Voronoi regions and the ratio between mean and standard are listed in Table 1.

Table 1 Metric information of the two maps in Figure 1 (The area of the map is a unit)

	H(M)	R_M	S_E	σ
Map in Figure 1 (a)	4.2848	80.51%	0.025	2.84%
Map in Figure 1 (b)	5.1260	96.32%	0.025	1.51%

From Table 1, it is clear that the map shown in Figure 1(b) contains higher metric information than that in Figure 1(a). Considering the fact that they should have the same amount of statistical information as pointed out in Section 2, it seems logic to claim that these measures is more appropriate than the statistical information.

4.2 Topological information: new vs old

The second test is on the topological information. Using the new index, the results for the two graphs in Figure 2 would be different. In Figure 2(a), there are seven vertices and the total number of neighbours for all vertices are twelve. The average number of neighbours for each vertex is 1.7. In Figure 2(b), there are seven vertices as well but the total number of neighbours for all vertices are fourteen. The average number of neighbour for each vertex is 2.0. It is then clear that Figure 2 (b) is more complex than Figure 2 (a).

Table 2 The average number of neighbours for Figure 3 and 5

	N_T	N_S	H_T
Figure 3(a)	40	206	5.15

Figure 3(b)	40	188	4.70
Figure 5	13	54	4.15

To further elaborate the adequacy of this new measure, the index value for the Voronoi regions shown in Figure 3 and 5 are also computed and listed in Table 2. It shows that the map shown in Figure 1(a) is more complex than that shown in Figure 1(b). This is because the three symbols are mixed into the building symbols.

4.3 Thematic information

The thematic information for the two maps shown in Figure 1 is also computed and shown in Table 3. It is very clear that the map shown in Figure 1a has more thematic information because the tree symbols are scattered around building symbols. On the other hand, the thematic information contained by the map shown in Figure 1b is lower because the three types of symbols are quite clustered. Therefore, the thematic information defined in this way seems very meaningful, as well.

Table 3 Thematic information of the two maps in Figure 1

	Thematic Information H(TM)
Map in Figure 1 (a)	28.2
Map in Figure 1 (b)	16.4

5. Information content of maps at different scales

After this evaluation, it seems that these new measures are very appropriate for measurement of information contents. In this section, the variation of information contents in maps with scale will be discussed.

It is understandable that a good example is a key to the meaningfulness of any experimental work. In this test, the example used in a test is adopted from the textbook written by John Keates (1989). There are four maps at different scales, one for 1:25,000, 1:50,000, 1:100,000 and 1:250,000. These maps and their corresponding Voronoi-diagrams are shown in Figure 6. The number of objects on these four 304, 82, 11 and 5.
The entropy of each map is then computed and recorded in Table 4. In other to compare the entropy values for maps at different scale, the normalised entropy is also recorded in Table 4.

Table 4 The entropy values of maps at 4 different scales

Scale	1:25 000	1:50 000	1:100 000	1:250000
Entropy	7.77	6.36	3.46	2.32
Normalised Entropy	0.94	0.93	0.77	0.73
Number of objects	304	82	11	5

It seems the normalised entropy value is stable for the first two and last two map scales. This is because during the generalisation, cartographers also try to keep good clarity of the map while trying to put as many symbols as possible on the map. But there is a jump between scales 1:50,000 and 1:100,000. This may indicate that there is a significant change in the representation on maps with scale variation from medium to small.

Table 5 Relationship between metric information ratio and scale ratio

Scale Ratio	Root Square of Scale Ratio	Entropy ratio	Object number ratio
50,000/25,000 = 2	1.41	1.22	3.71
100,000/25,000 = 4	2.00	2.25	27.6
250,000/25,000 = 10	3.16	3.35	60.8
100,000/50,000 = 2	1.41	1.84	7.45
250,000/50,000 = 5	2.24	2.74	16.4
250,000/100,000 = 2.5	1.58	1.49	2.2

Figure 6 (a) A map at 1:25,000

Figure 6 (b) Voronoi-diagram of Map (a)

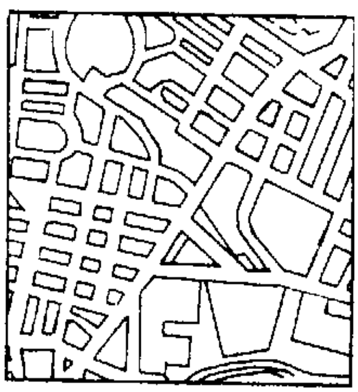


Figure 6 (c) A map at 1:50,000

Figure 6 (d) Voronoi-diagram of Map (c)

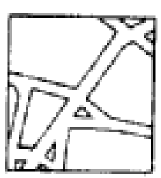


Figure 6 (e) A map at 1:100,000 Figure 6

Figure 6 (f) Voronoi-diagram of Map (e)

Figure 6 (g) A map at 1:250,000

Figure 6 (h) Voronoi-diagram of Map (g)

Figure 6 Maps at four different scales and their corresponding Voronoi-diagrams

However, if one takes the ratio of entropy values at different scale and also takes the root square of the scale ratios, then could find a general correspondence (Table 5), although there are deviations. But if one considers the ratio between number of objects and scales, which has been formulated as radical laws, the deviation is far bigger. This makes the authors to think of the general trend between the entropy ratio and scale ratio as follows:

$$\frac{H(S_1)}{H(S2)} = \sqrt{\frac{1/S_1}{1/S_2}} = \sqrt{\frac{S_2}{S_1}} \quad (15)$$

6. Conclusion

In this paper, existing quantitative measures for map information have been evaluated. It has been pointed out that these are only measures for statistical information and some sort of topological information but have not taken into consideration of the spaces symbols occupied and spatial distribution of symbols. As a result, a set of new quantitative measures is proposed, i.e. for metric information, topological information and thematic information. In these measures, Voronoi regions of map features play a key role, which not only offer metric information but also some sort of thematic and topological information. Experimental evaluation is also conducted. Results show that the metric information is more meaningful than statistical information and the new index for topological information is also more meaningful than the existing one. It is also found that the new measure for thematic information is also useful in practice.

For the metric information on maps with different scales, the ratio of entropy value is more or less linearly proportional to the root-square of the scale ratio. Of course, this conclusion is based on this limited results and more investigation into this matter is still very desired.

Acknowledgement

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