A STUDY ON THE MEASURING OF MAP ELEMENT

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In the relevant cartographic literatures there is not any close mathematical definition for map elements yet, Therefore, a study on quantitative description and measuring of map elements has been made by the authors. The research of the mathematical definition fo map elements and its measuring problem are presented in this paper.

1 Common characters of map elements

All map elements contain two facts of objective reality, i.e., the locality and its character. The locality of point is the first feature and it is related to the graphic structure. The second feature on the point concerns the substantial content of map element.[1]

Every map element *i* corresponds a qualitative characteristic function μ_i; *i* class map element has its image feature value *j* and at the same time, it has its certain ruling scale *t*. Therefore, the three features of *i*, *j* and *t* are common characters of all map elements. The computation formula of μ_i, *j* and *t* is as follows:

$$\begin{cases} \mu_i = {}^{i}\!/_{I} \,\big|\, i = 0,1,2,\ldots,I & \delta \in [0,1] \\ j = 0.25911\delta + 0.32469\eta + 0.4162\xi & \eta \in [0,1] \\ t \in [0,1] & \xi \in [0,1] \end{cases} \qquad (1)$$

Where *i* is the order number of map element in certain classification system I, the index set of i is I; the allocation of printng ink for obtaining value *j* of image feature, *i.e.*, the proportion of printing ink of yellow, fuchsin and blue, are δ, μ and ξ respectively; *t* is ruling scale.[2]

2 The metric space of map element and three equilateral value plane

2.1 Metric axiom and metric space

2.1.1 Metric axiom

Let R^n be the Euclidean space of n-dimension, *n* is the natureal number, *d* is the real number function on the $R^n \times R^n$ .if the following conditions are sufficed:

1) non-negative of *d*: $d(x,y) \ge 0$;
2) identical law: $d(x,y) = 0 \Leftrightarrow x = y$;
3)symmetric law: $d(x,y) = d(y,x)$;
4)triangle inequality: $d(x,z) \le d(x,y) + d(y,z)$

then *d* is called a metric of space R^n,the conditons 1)～4) are the metric axiom.

2.1.2 Metric space

Definition 1 Let x , y and z be the elements of set X, if there exists a function d: $X \times X \rightarrow R$ and it suffices the metric axiom, then the (X , d) is called as metric space and d is called variable or distance function on X, the all elements of X are called the points of metric space (X, d), the distance between x and y is called d(x,y), Any subset that belongs to X is called the point set of space (X, d).[3]

2.2 The metric space of map element

Definition 2 The space constructed with μ_i, j and t as the coordinate axes is called the metric space of map element. (Fig.1)

2.2.1 Qualitative feature's equi-value surface S_i

Definition 3 Qualitative feature's equi-value surface is the perpendicular plane of qualitative feature coordinate axis μ_i in the metric space, it is shown as S_i ,*i.e.*,$P_{i00}P_{i10}P_{i11}P_{i01}$ plance in Fig.1. let $S_{i=0}=S_{imin}$, it calls planiform characteristic plane.

2.2.2 Image feature equi-value surface S_j

Definition 4 The perpendicular plane of image feature coordinate axis j in the metric space is called the image feature equi-value surface, it is denoted as S_j, *i.e.* , $P_{0j0}P_{1j0}P_{1j1}P_{0j1}$ plane in Fig.1, let $S_{j=0}=S_{jmin}$,it is called the colurless plane.

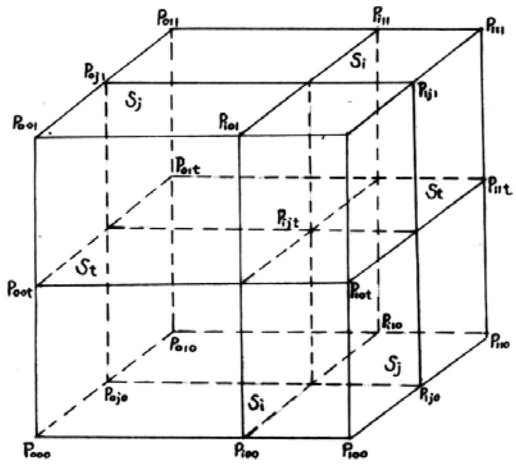

Fig.1 The Metric Space of Map Element

2.2.3 Level feature equi-value surfadce S_t

Definition 5 The perpendicular plane of ruling scale coordinate axis t in the metric space is called the level feature equi-value plane, it is denoted as S_t, *i.e.*, $P_{00t}P_{10t}P_{11t}P_{01t}$ plane in Fig.1. let $S_{t=0}=S_{tmin}$, it is called the white plane.

3 The measure of map element

3.1 The characteristic point P_{ijt} of map element

Definition 6 There exists $P_{ijt} \in X$,if it suffices

$$P_{ijt}=S_i \cap S_j \cap S_t \qquad (2)$$

then it is called the characteristic point of map element. Because of every map element X_{ijt} corresponds one bye one the characteristic point P_{ijt} of map element ,so between X_{ijt} and P_{ijt} there exists the equivalent relation. Therefore we can get the definitions as follows:

Definition 7 Areal map element There exists characteristic point of map element $P_{ijt} \in X$, if it suffices the condition $P_{ijt} \in S_{\text{imin}}$, then ,the areal P_{ijt} is called map element characteristic point and denoted as P_{ojt}, it corresponds the map element X_{ojt} and called the areal map element ,namely

$P_{ijt} \in S_{imin} \Leftrightarrow X_{ijt}$ is the areal map elenetnt X_{ojt}

In X_{ojt}, when $j \neq 0$,$t \neq 0$,it is the colour areal map element ;when j=0 ,t=0 , it is the white areal map element and called the X_{000}.

Definition 8 Point or linear map element There exists characteristic point of map element $P_{ijt} \in X$, ,if it suffices the condition $P_{ijt} \notin S_{i\,\min}$, then P_{ijt} is called the characteristic point of point or linear map element, it corresponds the map element X_{ijt} and is called the point or linear map element. namely

$P_{ijt} \notin S_{i\,\min} \Leftrightarrow X_{ijt}$ is the point or linear map element

In X_{ijt}, when $j \in (0,1)$ and $t = 1$, it is called the colour "reality" element of point or line and denoted as X_{ij1}; when $j = 1$ in X_{ij1}, it is the black map element and denoted as X_{i11}.

3.2 Distance fundction of map elements

Definition 9 Let P_{000} be the origin of coordinates in metric space (X,d) of map element ,the distance $d(P_{ijt},P_{000})$ between the characteristic point P_{ijt} of map element and P_{000} is called the distance function of map element X_{ijt} on X:

$$d(P_{ijt}, P_{000}) = \sqrt{\mu_i^2 + j^2 + t^2} \qquad (3)$$

where μ_i, j and t are obtained from formula (1).

Definiton 10 The second characteristic value of map element

The ratio between the distance function of map element and $\sqrt{3}$ is called the second characteristic value of map element and recorded as b_{ijt}:

$$b_{ijt} = d(P_{ijt}, P_{000}) / \sqrt{3} \qquad (4)$$

where $d(P_{ijt},P_{000})$ is obtained from formula (3).

Because of there exists $X_{ijt} \in X \Rightarrow (\mu_i, j, t) \in X$, and thus it caused $b_{ijt}=f(\mu_i, j, t) \in [0,1]$ only, so b_{ijt} may act as the measure of $X_{ijt.}$ According to this the definiton of map element can be derived:

Definition 11 Map element X_{ijt} The point set with the same second characteristic value b_{ijt} is called the map element X_{ijt}. *i,e,,*

$$X_{ijt}=\{ b_{ijt} | \mu_i = g(i), (\mu_i, j, t) \in X_{ijt}\} \qquad (5)$$

where the b_{ijt} is obtained from formula (4).

When i=0,the formula (5) expresses the area map element X_{ojt}, *i.e.*, ground element. When $i \neq 0$, the formula (5) expresses the figure element composed of points and lines.

4 conclusion

In this paper according to the metric axiom and the common characters of all map elements, the authors put forward the metric space of map element and some new concepts such as the characteristic point of map element, the second characteristic value, etc.,also derived the new definition of map element that based on the second characteristic value. Due to the measurable quality of the second characteristic value , so it made the related map element X_{ijt} be measurable, too.

Reference Literatures

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