RESEARCH NEEDS IN ANALYTICAL CARTOGRAPHY

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ABSTRACT

Analytical cartography has been a growing subdiscipline of cartography for the last several decades. As such it exists at the intersection of quantitative cartography, geographic information science, discrete mathematics, computer and information science, and image analysis. A recent article by Moellering (2000) defines the scope and conceptual content of analytical cartography. The work explores and discusses the large number of active research areas currently going on in analytical cartography. This paper will investigate the research needs that flow out of this multifaceted set of research areas.

I. INTRODUCTION

Analytical Cartography has been a growing subfield of cartography since the 1960s when it was founded and initially defined by Tobler as a more mathematically and scientifically refined approach to cartography utilizing the idea of "Solving cartographic problems" (Tobler, 1966). Since that time, modern Analytical Cartography has expanded its scientific horizons such that it lies at the intersection of the fields of quantitative cartography, geographic information science, discrete mathematics, computer and information science, and image analysis. This view is an expansion and extension of the idea originally proposed Nyerges (1980). The goal of this paper is to investigate the research needs that flow out of this multifaceted set of research areas that are part of Analytical Cartography.

The beginning point of this discussion is based on the fundamental concepts of Analytical Cartography, the first of which is geographic map transformations (Tobler, 1961, 1986) which was the conceptual beginning point of the field. The second fundamental concept of that of real and virtual maps, and their 16 transformations between these four states of maps (Moellering, 1977, 1980, 1984). This basic concept explains and helps one understand various kinds of cartographic and spatial data operations such as digitizing, computerized visualization, spatial database processing, and also conventional cartography. The third fundamental concept is that of deep and surface structure in cartography as defined by Nyerges (1980, 1991). This idea was adapted from structural linguistics over to cartography where surface structure is the graphic visualization of spatial and cartographic data, and the deep structure is the deeper mathematical analysis of spatial data in a nongraphic domain. When the mathematical analysis is completed, many facets of the work then usually can be visualized. Fourier parameters are a good example of deep structure spatial information. The fourth fundamental concept is that of Nyerges Data Levels (Nyerges, 1980) which are: Data Reality - essentially the real world and the entities in it, Information Structure - a formal model of entities that exist in the real world, Canonical Structure - a data model of spatial objects that are a digital representation of the spatial entities above, Data Structure - a structured organization of spatial data into forms such as raster, vector or relational forms, Storage Structure - essentially the file structure that implements a particular data structure, and finally, Machine Encoding - which is essentially how the basic bits and bytes of computer code are mapped and stored in computer hardware. Spatial scientists and cartographers typically deal with the top four Nyerges Data Levels, while computer scientists work with the lower two levels. A fifth fundamental concept is that of spatial primitive objects. These digital spatial objects such points, nodes, line segments, chains, polygons, pixels, grid cells and others form the basic set primitive spatial objects. These primitive objects then can be built up into higher level complex objects such as networks, surfaces, spatial data layers. Many national and international spatial data standards specify such sets of spatial primitive objects. The sixth fundamental concept in analytical cartography is that of the Sampling Theorem, which relates spatial sampling intervals in 1-, 2- 3- or n-Dimensional spaces, and mathematically relates the sampling rate to spatial resolution. From these fundamental concepts in analytical cartography flow many of the more specific concepts in the field. These and many other concepts that define the scope and conceptual content of Analytical Cartography are reviewed by Moellering (2000) in a dedicated issue of <u>Cartography and Geographic Information Science</u> titled "The Nature of Analytical Cartography".

With the large amount of research work relating to Analytical Cartography that has been carried out over the last several decades, a fairly large set of concepts have evolved in the field as discussed in the reference above. However, this body of spatial theory is still being developed and extended with further new research. From overviews of this and related work have come research needs statements to facilitate further research in the field. Moellering (1994), after heading the U.S. national effort of develop the Spatial Data Transfer Standard, saw many areas where further spatial research was needed in the field. The article showed that there are many gaps in the spatial theory that one needs to utilize to define effective spatial data transfer mechanisms that extend out to many areas of Analytical Cartography and the related spatial data sciences. More recently Mark (2000) examined a broader set of cross-disciplinary research issues involved in Geographic Information Science, such as integration, scale, process models and usability, that result in a set of research challenges of representation, uncertainty, cognition, and simulation.

This paper will discuss the research needs for most of the important areas of the field of Analytical Cartography. As such, the discussion will investigate the research needs that flow out of this multifaceted set of research areas in Analytical Cartography referenced above. In a short paper such as this, one must necessarily be rather general, and usually does not have the luxury to discuss these topics in detail.

II. RESEARCH NEEDS IN SELECTED AREAS

The following are a set of selected research areas where various research needs can be seen. By the nature of such a paper, only a summary discussion can be provided.

Geographic Map Transformation

Much of the earliest work by Tobler in Analytical Cartography concerned geographic map transformations. Out of this work resulted mathematical cartograms and related transformations. Extensions of this work can be envisioned in the area of map and data structure transformations, algebraic geometric data transformations, map conflation, transformations that result in maps with different topology, and the view of map transformations organized along the lines of their geometric and attribute characteristics. Some work has already be done on these and related topics, but more work is needed.

Real and Virtual Map Transformations

Moellering developed the concept of real and virtual map transformations where the state of the Real Map and three states of Virtual Maps conceptually interact with each other to produce 16 transformations between the various kinds of real and virtual maps. As such, these kinds of transformations specify all of the various kinds of data transformations found in cartography and the larger spatial data sciences. These transformations also provide much insight into how computer-based systems process spatial data as it flows through such spatial data systems.

At the present time such Real/Virtual map transformation are specified descriptively in such data processing flows. What is needed is a more rigorous algebraic formulation that expresses these transformations in a deeper form. Nyerges has related these transformations to deep and surface structure, and to his Nyerges Data Levels. Now more mathematically rigorous forms of these relationships are needed.

Spatial Primitive Objects

Spatial primitive objects such as points, nodes, line segments, links, chains, polygons, pixels and grid cells have been fairly well developed in the 0-, 1-, and 2-D spatial dimensions. These objects possess various combinations of geometric and topological characteristics. These primitive spatial objects are used to construct higher level complex objects such as networks and various kinds of surfaces.

However, three dimensional spatial primitives are relatively undeveloped at this time. There is a great need to develop such three dimensional primitive objects for digital spatial data systems. One of the major bottlenecks in such developments is the deeper understanding of the topological relationships inherent in such objects. Some of the needed topological theory already exists in the mathematical field of topology. One task is to bring this topological theory from mathematics over to Analytical Cartography and the other spatial data sciences so these 3-D primitive objects can be specified in a systematic and coordinated way. One example of this need is for a comprehensive specification of irregular 3-D polytopes. Volumetric processing of spatial objects depends on this theory for the field. There is a possibility of topology-only primitive objects as well. Currently, most primitive spatial objects that are widely used have both geometric and topological characteristics. It some instances one can see a need for topology-only objects. More research is needed.

The temporal dimension must also be considered. Time is just as much a part of the Real World as are the three spatial dimensions, and geometry and topology. Currently, most systems treat time and temporal properties of spatial data separately from the primitive spatial objects themselves. However, one must consider the possibility of temporal characteristics of spatial objects being integrated in with the geometric and topological characteristics of such objects. This work is still in a rather early stage of research, and much more research is needed.

Spatial Semantics

One can view spatial data operations from the view of syntax and semantics. Syntax has to do with the digital encodings of spatial data. Semantics has to do with meaning. In the spatial world, semantics has to do with the meaning of spatial entities in the Real World, digital spatial objects and the relationships between them. A much more specific use of semantics comes in the process of spatial data transfer between systems, and integrating diverse spatial variables together into the same spatial database. If computer-based systems are to successfully be able to combine spatial variables together in a spatial database, then the definition and meanings of the definitions of the variables must be the same, or at least compatible with each other. The concept of spatial metadata was developed to provide additional information about spatial databases, and for computer-based systems to successfully process metadata from different databases together, the semantics of the definitions must be compatible. Much further research is needed in the area of semantics in general, but specifically in the areas of metadata compatibility, and the area of the basic relations between entities in the real world, and area called ontology.

Sampling Theorem

As discussed above, the sampling theorem specifies the relationship between the spatial sampling distances on various dimensional objects, and the spatial wavelength that can be resolved from that sampled data. The equation is Delta = 1/2 Lambda, where Delta is the sampling distance, and Lambda is the spatial wavelength. This concept is closely related to the notion of spatial resolution. The use of the sampling theorem in the square regular cellular domain has been well established in 1-, 2- and 3-D objects for many years. However, the situation in irregular spatial objects is quite different, and not as well developed. Tobler (1984) provided a way to estimate the Delta, the estimated sampling interval from a set of irregular polygons, called resels, in the 2-D domain. More recently, Tobler (2000, p. 193.) extended this estimation process to N dimensions to provide what he calls Average Spatial Resolution. Now what is needed are wider investigations into the use and application of Average Spatial Resolution in Analytical Cartography and the spatial data sciences in general.

Spatial Frequencies

The concept of spatial frequencies is very scientifically appealing, especially as utilized with spatial surfaces such as topography. If the original data exist in simple square cell form, Fourier analysis can be utilized in a mathematically rigorous way to calculate the spatial frequencies contained in the surface. This can be done in terms of the frequency, wavelength, and amplitudes of the spatial frequencies involved, and they can be portrayed in spectral diagrams of the results. Further spectral components like the variance spectrum can be calculated and analyzed.

The problem is that the above sorts of spatial spectral analysis, and the relating spatial frequencies can, in general, only be calculated for data in the regular cellular domain. The equations brought over from mathematics to analyze the regular cellular data cannot be used to analyze spatial data in irregular cellular systems. The only exception is that of Moellering and Tobler (1972) where they were successful in calculating the variance spectrum for irregular cellular data that were contained in an existing hierarchical structure. What is needed here is to extend the mathematical theory and equations such that one can calculate the spatial frequencies, their individual frequency, wavelength, amplitude and other resulting components directly in irregular polygonal systems of data. Accomplishment of a general solution to the calculation of spatial frequencies in the irregular domain would be a great step forward in Analytical Cartography and the spatial data sciences in general.

Spatial Neighborhood Operators

Most such neighborhood data operators work mathematically in the regular square cellular domain, although such operators can work on regular intervals of linear data. It is also possible to use such regular cellular operators in 3-D with volumetric cubic data cells, but this possibility is not used much in Analytical Cartography. The most common use of such neighborhood operators is in the 2-D square cellular data domain where operations such as smoothing, edge enhancement, variance estimation, slope calculation and many other operations can be accomplished with parametric interval/ratio data. It is even possible, in some cases, to do smoothing in the square cell domain with nominal data, such as land use.

The most important research need is to extend the working of such neighborhood spatial operators into the irregular cellular domain of irregular polygons. Much thematic spatial data is collected in irregular data polygons such as census tracts, and having a set of well specified neighborhood operators for such irregular polygonal structures is necessary for the field. To date some research has been invested to develop such irregular neighborhood operators, and they can be used in some instances, but a much more comprehensive and systematic specification of such operators is really needed. Tobler's Average Spatial Resolution is a step forward to estimating the resolution in such irregular cellular systems. However, more research is needed to develop irregular neighborhood operators further on a more systematic basis, and to relate them directly to the spatial frequencies that are components of the surface being analyzed.

It turns out that most spatial neighborhood operators process spatial data in a specific data structure. There are many kinds of spatial data structures, raster and vector being two of the most basic kinds, and most spatial data operators, as specified mathematically, only work in one kind of spatial data structure. This means that in most cases when one then begins to run operators that do the same thing, smoothing for example, in different spatial data structures, then one has to rewrite or rework the equations, and perhaps the weight fields, to run in the neighborhood operator on the new data. What is really needed are more general neighborhood operators that are specified such that they work independently of the spatial data structure(s) involved. Data Structure independent spatial neighborhood operators would be a wonderful step forward. Now the research is need to begin to accomplish that goal.

Information Theory

Information theory is a field that was originally developed in signal processing. In many cases this concept is employed as an entropy measure for some kind of spatial distribution. Entropy is usually calculated as a parameter H that is associated with some kind of spatial distribution that is being optimized. The big conceptual advantage of entropy measures is that they are distribution free statistics, as contrasted to least squares measures found in surveying, geodesy, and geography which utilize a Gaussian statistical distribution. It can be shown that many spatial statistical Z distributions to not have a normal Gaussian distribution, and if not corrected, can bias the results of the statistical analysis. Entropy statistics are distribution free by their nature and definition, meaning that there is NO assumed underlying base statistical distribution. Therefore the resulting entropy statistic, usually H, cannot be biased by a non-Gaussian or other unusual underlying statistical Z distribution. Entropy analysis has been used in the spatial data sciences to advantage and a little in Analytical Cartography. It has much more potential be utilized in distribution free spatial models in the field, and should be investigated much more thoroughly.

Fractals

Fractional Dimensional spatial objects, known as Fractals, have arisen from the mathematical concept of the Hausdorfer-Besicovich fractional dimension of an object. This concept was popularized by Mandelbrot in mathematics and picked up in geography and Analytical Cartography in the 1980s and 1990s. Lam and DeCola (1993) provide a good summary of work in their book. Most spatial objects as reckoned as line 1-D, area 2-D, or volume 3-D objects that have integer dimensions. The Fractal concept recognizes, for example, that as a line becomes more complex, its fractional dimension begins to increase as an decimal dimension, e.g. 1.35, and as the complexity of the line increases, the fractional dimension increases as well. At the limit, a line could have a maximum fractional dimension that begins to approach 2 because the extremely complex line begins to fill an area. Similarly, an areal object at it simplest may exist as a flat 2-D object. As it increases in complexity with peaks and pits becoming more pronounced, the factional dimension increases to say 2.2, 2.5, 2.7. At the theoretical limit the maximum dimension for a surface may begin to approach a fractal value of 3.

Fractals can be used as an analytical measure to measure the dimensional complexity of lines or surfaces in terms of their fractional dimension. They can be used to analyze various kinds of networks and spatial surfaces, as well as topography and some kinds of geological structures. They can also be used to generate artificial complex curves and surfaces. So far, one of the wider uses of fractals in cartography as an analytical measure for ascertaining line complexity under generalization. It can also be used to assess surface complexity that results from surface generalization. The fractal concept offers a potentially wide array of uses in Analytical Cartography and should be explored more fully.

Critical Surface Features

One of the more authoritative ways to analyze spatial Z surfaces is to ascertain and define the mathematical critical points and lines on the surface,where the first and second spatial derivatives change. Warntz (1966) has defined these kinds of spatial Z surface features as: peaks and pits, ridges and valleys, and passes and pales, all places on a spatial Z surface where the derivatives change. A careful identification of these critical points and lines can then be used to assemble them together in a topologically systematic manner. They can then be used to construct the topological skeleton of the surface. This network of critical points and lines that form this topological skeleton of a spatial Z surface is called a Warntz Network, after William Warntz, who first proposed the concept.

The Warntz Network can then form the fundamental topological structure of a spatial Z surface. It has been used as an analytical tool to assist in the calculation of slope/gradient, line-of-sight, visibility and many additional characteristics of a surface. The theory seems to be rather well defined in topology, but applications in the spatial data sciences have yet to assemble a full theoretical mathematical definition of a Warntz Network of a spatial Z surface. Meanwhile many applications use the theory in its current form. What is really needed is to finalize the topological mathematical definition of a Warntz Network such that it is complete and with no theoretical gaps in it. If that can be accomplished, then a researcher could utilize the Warntz Network for many uses with the confidence that the theory is robust and complete. New uses could flow from this development.

Polygon Overlay and Analysis

Polygons and polygon analysis by its nature is part of the irregular spatial domain. In the early days, most of the work on polygons was accomplished by interpolating back to regular square cellular structures because the direct polygon operations in the irregular domain hardly existed. In recent decades research has developed analytical approaches to define point/area spatial duals with Thiessen polygons and Delunay triangles. Direct overlay of irregular polygons has improved in that research has revealed that there are least 34 different topological cases of how such polygons can overlay each other. These findings of such a large number of topological cases of how polygons can overlay each other reveals rather clearly why the early polygon overlay algorithms failed on a fairly regular basis. The reason is that these algorithms only considered the major topological cases and missed the more esoteric cases that do not occur very often.

Many polygon overlay processes utilize the concept of the Least Common Geographical Unit (LCGU), where all levels can be overlayed, and the smallest subpolygons are defined as LCGUs. These LCGUs then form the basic building blocks from which spatial z variables can be transformed from any one of the polygonal systems involved to another. Further research is needed to refine these processes such that they can handle all of the possible topological overlay cases, 34 or perhaps more, in a complete and mathematically authoritative way. Inside of each polygon, the spatial variable is usually assumed to be uniformly spread throughout the polygon, so that when it is split up into the LCGUs, and reassembled in to another polygonal system, the distribution of that Z variable will be correct for the new polygonal system. It is clear that this Z variable uniformity assumption is rarely met in a rigorous way, and that research in need to further investigate this problem. If one views the polygons with their respective Z values, one can look at the situation as a discrete Z surface for which one can analyze and evaluate the volume of the collective set of polygons. Robinson suggested this consideration several decades ago, and more recently Tobler introduced this concept in a more mathematically sophisticated way under the concept of pycnophlacticity. Tobler has used this pycnophlactic mathematical property in a rather elegant way under certain conditions to transform discrete Z surfaces to continuous surfaces. What is needed now is to carry on to extend these fundamental threads of the spatial polygon theory to produce new and more integrated topological and mathematical solutions.

Shape Analysis

The quantitative measurement of shape has always been a difficult concept to approach analytically. Shape includes aspects both aspects of topology and geometry. Most single parameter measures result in being measures of compactness, only one characteristic of shape. Many of these single parameter measures are deeply flawed conceptually because their units do not cancel. Since that time many efforts have been made to develop multiparameter analytical measures of shape. These usually involve the quantitative assessment of the outer boundary of the object being analyzed. They include several piecewise measures, the best of which include a syntactical organization of the structure of the shape. There have been many analytical approaches developed that include various method of moments measures in two and three dimensions, and various analytical approaches based on the Fourier analysis of different kinds of intrinsic functions. There is a need to extend this research to find improved methods because areas like generalization and feature identification utilize some of the basic theory from shape analysis in their implementations.

Spatial Data Structures and Models

Spatial data models have seem some dramatic developments since their beginnings as geometry only vector data structures in the 1960s. Later in that decade theorists came to realize that topological properties like connectivity and contiguity were extremely important characteristics that must be included in most vector data models at Nyerges Data Levels 2, 3, and 4. The 1970s saw the development of a wide range of data models that included geometry and topology as basic characteristics of the primitive spatial objects included as components of such models. Here one saw the development of the Dual Independent Map Encoding (DIME) model developed by the U.S. Bureau of the Census, and the Triangulated Irregular Network (TIN) models to mention some of the best known examples. In the 1980s the spatial data sciences saw the rise of raster data models related to satellite imagery and square cell models such as the U.S. Geological Survey Digital Elevation Models (DEM). These kinds of spatial data models are basically geometry only models, but now with special isotropic regular cellular characteristics of the same size, shape, number of neighbors and similar characteristics. The 1990s has seen the rise of relational spatial data models which focused on the relationships recognized in the real world, abstracted into a formal Nyerges Level 2 Information Model, and then carried deeper into Nyerges Levels 3 and 4. Throughout the 1980s and 1990s many researchers sought to develop hybrid spatial data models that included characteristics from more than one family of spatial data structures. The Vaster data model is one that tried to blend together aspects of the vector and raster models, which in absolute terms succeeded, but in the end included the overhead of both structures which was cumbersome and unwieldy. Others have tried to employ dual models together, such as vector and relational models working in a sort of parallelism. Some of the popular GIS systems utilize this approach.

Some researchers sense that there may be more synergism to be found in these sorts of hybrid models, or perhaps a new form of spatial data model could emerge that has characteristics and capabilities of more than one family of spatial data models. This would demand a higher level of theoretical insights than currently realized, but efforts must be made to extend these concepts.

Map Generalization

Map generalization as a task of cartography and spatial visualization utilizes many of the fundamental spatial theories discussed above, such as resolution, geometry, topology, fractals, linear and areal neighborhood operators, surface features, shape analysis and so on. In the early days most cartographers saw the map generalization task as the controlled reduction of information in the graphic realization of the map, or surface structure. More recently, a few researchers have come to realize that the map generalization task may include analytical/mathematical work in the deep structure before being visualized graphically in the surface structure. One such characteristic that entered consideration in recent years is that of the topological properties of various kinds of spatial structures. It is clear to this researcher that more such deep structure analytical approaches could be brought into map generalization activities. A more active effort should be made to search out and develop such theory that might be applied to the map generalization problem.

Analytical Visualization

To most individuals in the field, this term might be viewed as an oxymoron, in that the two components seem to conflict with each other semantically. Actually, the term analytical visualization was coined by this author as a way of expressing the strategy of visualization as being more than graphic realization, but could include deep structure analytical theory in with the visualization task. Such theory could include quantitative theory as well as color theory. One could also include spatial data analysis and modeling as an applied part of this effort, but the main thrust of this work can be characterized as developing strategies where deeper theory is utilized to achieve a better and perhaps more effective visual result. Digital stereoscopic visualization is one example, because the mathematical parameters of the process can be adjusted to create various visual effects. Here such theory can be combined with the massive technological strides in computer display hardware that have been made in recent years to result in some exciting developments in this research area.

III. A GRAND THEORETICAL CHALLENGE

As one works though these various areas of spatial theory in Analytical Cartography, one wonders if there may be some much larger theoretical questions that should be answered. One of the deepest and grandest research challenges is whether it is possible that a general theory of spatial data might exist, in a manner similar to general theory in physics as originally suggested by Moellering (1994). In the 19th century, much time was consumed by physicists debating the advantages and disadvantages of the particle theory of light versus the wave theory of light. Some problems of light could only be solved by one theory and not the other, and visa versa. A few problems could be solved by both theories of light. This continuing debate in the field of physics raged on for several decades during the later half of the 19th century. In the end, a unified theory emerged which showed that the particle theory of light and the wave theory of light are special cases of a more general theory.

One wonders whether the situation in cartography and the spatial data sciences in the 21st century is similar to physics in the 19th century. Here we have two major spatial data structure types, raster and vector which have been competing with each other to see which can handle spatial data more effectively. Similar to light theories with physics, some operations that can be carried out straightforwardly in vector structures, such as topological networks, can be only done with extreme difficulty with raster structures. Similarly, spatial frequencies are easily calculated in raster square cell structures with Fourier analysis, and can only be calculated with in one instance in the irregular vector cellular domain. Is it possible that vector, raster relational and perhaps object-oriented data structures are special cases of a more general theory? The answer to this question is not known, but it is clear that the emergence of such a theory would greatly advance many of the research problems discussed above. It would provide a more advanced theoretical base for the these areas. In the United States, a group of researchers in Analytical Cartography is contemplating these possibilities and the general situation involved.

IV. SUMMARY AND CONCLUSIONS

This paper is a survey of research needs in the field of Analytical Cartography. Of necessity, it is not possible to go into great detail with a paper of this length. It began with a view that Analytical Cartography is scientifically located at the intersection of the fields of quantitative cartography, geographic information science, discrete mathematics, computer and information science, and image analysis. It is seen that the field is based on the fundamental concepts of geographic map transformations, real and virtual maps, deep and surface structure, Nyerges Data Levels, spatial primitive objects, and the sampling theorem.

From the fundamental concepts in the field spring a large variety of research areas that include geographic map transformations, real and virtual map transformations, spatial primitive objects, spatial semantics, the sampling theorem, spatial frequencies, spatial neighborhood operators, information theory, fractals, critical surface features, polygon analysis and overlay, shape analysis, spatial data structures and models, map generalization, and analytical visualization. This list comprises many of the research areas active in the field, although one could cite many more detailed research areas of interest.

From these, one can see a grand research challenge emerging of asking whether it is possible that there exists a general theory of spatial data. If this is true, such a general theory could serve to theoretically connect many of these areas together in a closer conceptual way. Such a grand spatial theory would be the "holy grail" of the spatial data sciences. However, it is clear that a very concerted research effort would be required to develop this concept, if it is possible.

Taken together, these research needs provide a formidable research agenda for Analytical Cartography and related areas of adjoining sciences. They also provide a wealth of research opportunities for researchers, especially young investigators entering the field. The future of the field is bright with this wide variety of research needs, which poses a significant challenge to current and new researchers in the field. The notion of the possibility of a general theory of spatial data provides a shining goal towards which the leading researchers can strive to develop.

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