A NEW CARTOGRAPHIC GEOMETRY

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The standard model of the geometry of cartographic visualization in geographic information systems (GISs) is based on the classification of cartographic objects into points, lines, and polygons, represented with zero-, one-, and two-dimensional symbols. This is restrictive because an object or symbol may actually span more than one of these dimensional categories or may occupy an intermediate position between categories. A more complete, "cognitive" model of the geometry of visualization is proposed here. The new model is more flexible because it permits an object to be positioned anywhere within several continua between the standard dimensional categories.

The geometry of cartographic visualization differs from classical geometry because while the latter involves abstract objects--points, lines, and regular polygons--the former is based on the human geometric classes that one uses to describe objects in the real world, which only rarely have perfect (classical) geometrical forms. Such reasoning has led to topological data structures (Burrough, 1986; Laurini and Thompson, 1992), in which a polygon is any bounded two-dimensional object with an interior, a line or arc is any curvilinear feature from a straight line to a winding stream, and a point feature is anything that appears as a point or point symbol on a map, from a tree to a city. In contrast, the geometry of thematic mapping symbols is closer to the classical ideal, because cartographic symbols and patterns tend to be regular or based on regular forms, such as circles, squares, rectangles, and straight lines. Exceptions occur when the visualization technique is in some way directly dependent on the shape of map objects, as in the case of cartograms.

The current geometric model attained its standing with the spread of GISs. In adapting maps to computers, it has been necessary to simplify cartography in many ways. Most graphic images on a computer screen are composed of points, lines, and polygons, whatever they may represent in real life. GIS software packages invariably include a smaller number of techniques, fewer different symbols, and fewer options for text placement than are available to manual cartographers. Since GISs tend to define all cartographic objects in terms of points specified by coordinates and figures created by linking points together, there has been a move toward seeing cartographic objects in what might be called "graphic integer" format--having dimension 0, 1, or 2 (sometimes also 3). This standardization has led to a decline in the accuracy of thematic maps, partly through poor choice of mapping techniques and partly because GIS software does not permit enough variation in symbol characteristics to allow cartographers to capture the true dimensional character of geographic objects or data.

Geometric Models for Cartography

Previous researchers have recognized aspects of the dimensional problem. Hsu (1979) makes an important distinction between the geometry of information used to create maps and the geographic arrangement or "distributional form" of these data. One can observe thematic information at different geometrical levels or "spatial dimensions": points (tree or building locations), lines (roads, canals), areas (land cover). The elements of standard cartographic topology--points, lines, and polygons--correspond directly to these four spatial dimensions. However, when creating a map, one does not always use symbols of the same spatial dimension as the information. For example, the only truly accurate and precise way to map the distribution of a population is by means of point symbols, with one per person and thematic characteristics shown by means of variation in the color or form of the symbol representing each individual. But census agencies do not release the locations of individuals, and cartographers must instead make use of data aggregated to enumeration units. Choropleth maps of enumeration-unit-based data, very common in demographic mapping, are in a sense imperfect representations of what are still point data. Choropleth symbols are still punctual because each is derived from a single value, generally some composite of the characteristics of the individuals who live there. In contrast, a land cover map showing forests and deserts is truly areal: areas colored to represent land covers cannot be shown by graduated circles without loss of information (the shapes of the land cover regions), whereas one can substitute a point symbol for a choropleth with no loss of information.

Thematic cartographers often must choose symbols based on distributional form, rather than the spatial dimension, of their data. Shiryaev (1987) and Hsu both distinguish between discrete point measurements and continuous information covering the map area, while MacEachren and DiBiase (1991) add that data may vary smoothly in space or change abruptly at boundaries. For example, tax rates are continuous but abrupt, because they change at political boundaries, while trees are discrete but may be smoothly distributed throughout a region. Not all data fit neatly into one of the four distributional forms; both Shiryaev and MacEachren and DiBiase present their models as continua between endpoints.

Similarly, not all cartographic symbols fit exactly into the standard topological classes. Keates (1973) presents several examples of symbols that do not fit clearly into the conventional categories because their forms are scale-dependent. For example, cities may be depicted either with squares of uniform size or by coloring the area that is included within its actual political limits. In the former case, the square is clearly a point symbol, because its form is unrelated to the city's true shape, while in the latter case one has an area symbol. Czerny (1993) adds that one must consider whether a symbol is "iconic", or in some way symbolic of the form or appearance of the real object it represents. A choropleth map of land cover that employs green for forest and tan for deserts is iconic because the colors are approximately true, while the use of blue for deserts would be clearly unrealistic and therefore non-iconic.

Cognitive Geometry

A new model has been developed by borrowing a concept from the theory of cognitive linguistics (Langacker 1987 and 1991; Geeraerts, 1988). Some linguists argue that categorizing words as parts of speech is impossible because many words have multiple functions or do not clearly fit into any of the standard categories such as noun or verb. Some words are "prototypical" examples of their categories--that is, they are always and indisputably nouns or verbs. The English word "atlas" can be considered a prototypical noun because it nearly always refers to a specific object, a book of maps, and has no other meaning or application. Other words fit into multiple categories or cannot be assigned to any of the standard classes. For example, the word "map" can act either as a noun or a verb, and in certain circumstances can have a meaning that is neither purely noun nor purely verb. If one creates a "map" of a part of the earth onto a sheet of paper, one both performs an action (the mathematical process of mapping points to paper) and creates an object (the resulting image).

Similarly, some cartographic objects are clearly zero-, one- or two-dimensional, while others do not fit clearly into a single dimensional category. For example, an unadorned fine line indicating a political boundary is clearly a one-dimensional symbol, because it has no interior structure and it represents something that has no width. In contrast, the linear symbols for highways on road maps are normally called one-dimensional, but they may also have some two-dimensional aspects, such as width or a colored interior. What is represented (roads) are also two-dimensional in reality, even if they are typically represented by means of "line" symbols. The political boundary is therefore a prototypical line symbol while the highway symbol is not, yet in the standard geometry of cartographic visualization, both become one-dimensional "line" symbols.

The dimensionality of cartographic objects in the new model is evaluated by looking at three factors: the form of the cartographic symbol, the form of the real-life object(s) that are being represented, and the visual variables used to display variation among objects. Analysis of a geographic data set with reference to the model can help determine which cartographic techniques are most appropriate for mapping. This in turn leads to maps that are more correct representations of reality.

The Cognitive Model

In contrast to the standard topological model, in which objects or symbols are classified as points, lines, or polygons, the cognitive approach envisions a continuous "symbol universe" without pre-ordained divisions into geometric categories. One can choose to define any region of this universe as the prototype of a symbol class (dark gray circles in Figure 1). Symbols that have some characteristics in common with a prototype but also differ in some important way form a non-prototypical buffer or "tidal zone" around the prototype "island", indicated as a light gray band in Figure 1. Finally, objects that are equally similar to two prototypes can be placed in an intermediate zone or "channel" between the two, while objects that have nothing in common with any prototype are either far out in the geometric ocean or may be designated as the prototype of another symbol class.

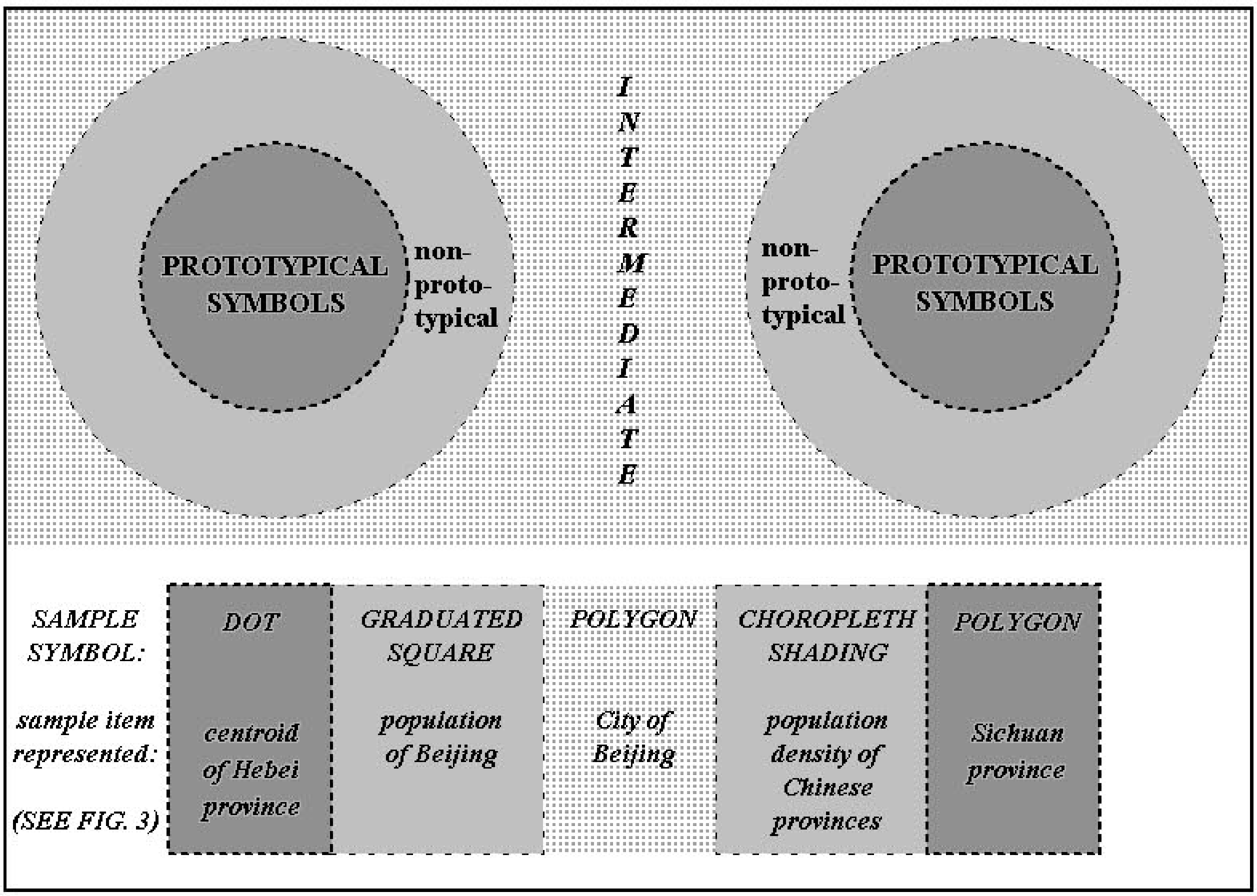
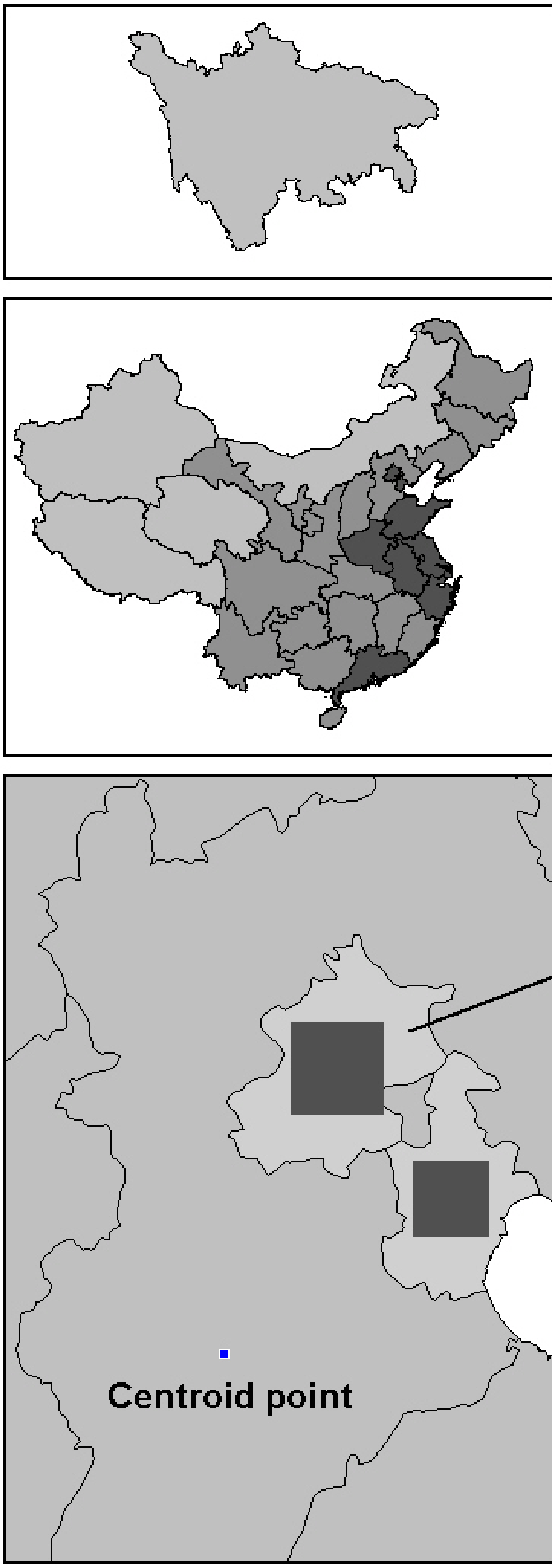


Figure 1. New Model of Cartographic Geometry

Figure 2 illustrates this range of possibilities. Sichuan province is a prototypical polygon, represented by a prototypical polygon symbol: it occupies area, cannot reasonably be displayed with anything but an areal symbol. A choropleth map of the population density of Chinese provinces must make use of



PROTOTYPICAL POLYGON

Sichuan province

* * *

NON-PROTOTYPICAL
POLYGON SYMBOLS

Choropleths
(population density)

* * *

INTERMEDIATE
POLYGON -- POINT

Beijing municipality

* * *

NON-PROTOTYPICAL
POINT SYMBOLS

Graduated squares

* * *

PROTOTYPICAL POINT

Centroid of Hebei province

Figure 2: Interdimensional Symbol Continuum

polygonal symbols but is really displaying zero-dimensional information, namely single density values for each province. The city of Beijing is in an intermediate position, because depending on the scale, it may be reasonably be displayed as either a point or a polygon. If the graduated squares representing Beijing and Tianjin are scaled in proportion to municipal land area, they are clearly non-prototypical point symbols, because their area has areal meaning yet they are point symbols without any tie to the actual form of the objects being represented. On the other hand, if they are scaled in proportion to population, they are closer to the punctual prototype--though still not as clearly prototypical as the centroid point of Hebei province, which is truly zero-dimensional. Each of the illustrations in Figure 2 may be matched to its position in the theoretical framework by reference to the lower half of Figure 1.

Prototypicality of Symbols

Definition of geometric categories in the cognitive model is akin to the creation of numerical data ranges in thematic cartography. One can create any number of sets of ranges from a given set of data, but certain sets of ranges will be particularly useful for specific purposes. Similarly, one is free to define any symbols or cartographic techniques as prototypical, but that is not to say that all geometric frameworks will be equally useful. In Figure 3, the standard topological model has been used as the starting point for

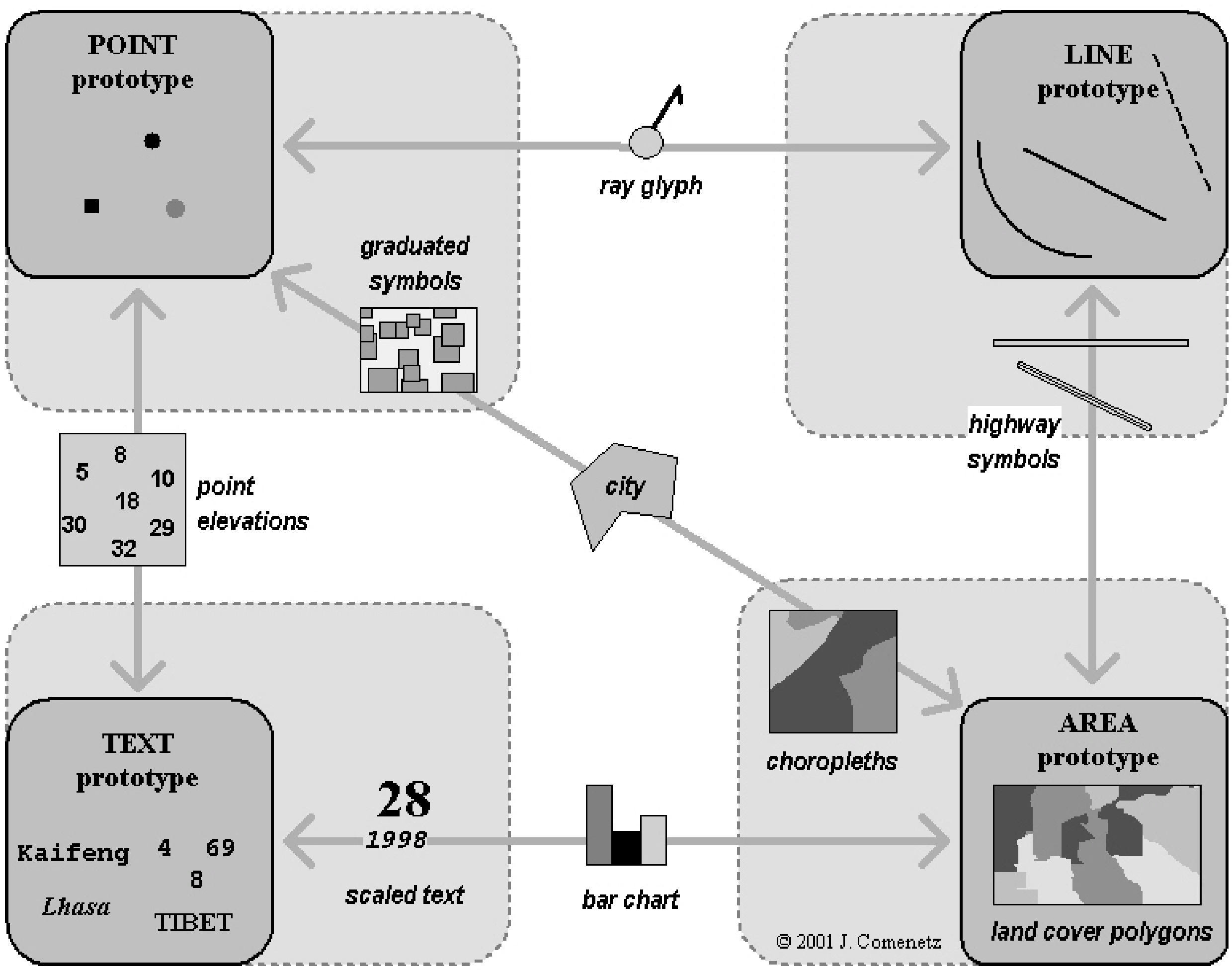


Figure 3: A Cognitive Framework of Cartographic Geometry

one possible cognitive framework for the geometry of cartography. As in Figure 1, prototypical symbols are shown as darker islands surrounded by lighter bands of less-prototypical symbols, with the uncolored background representing the intermediate zones between prototypical classes. Connecting arrows represent the theoretical continua between each pair of prototypes. (The five steps in Figure 2 can be matched to the continuum from Point Prototype to Area Prototype.) For example, dots representing points of known elevation are prototypical point symbols, while the word "Kaifeng" is purely a text label and has no geometric dimension. However, when one prints numerical values on a map to indicate elevations at specific points, the resulting "symbols" fall in the intermediate zone between points and text, incorporating elements of both.

The above distinctions are important because it becomes more difficult to select the best cartographic technique for a specific application as one moves away from prototypicality. Indication of the centroid of a polygon is best done, and can only be done, by means of a simple point symbol: a cartographer has no reason to doubt his or her choice of technique when using a prototypical symbol in a prototypical way. A centroid and the corresponding point symbol are both undoubtedly zero-dimensional. In contrast, when data do not have an obvious dimension--when they are not prototypically of a specific dimension--the situation becomes more complicated. For example, population data are either zero- or two-dimensional, depending on how they are reported, the scale of observation, and the level of detail. Published cartographic responses to the problem of mapping one of the most basic demographic variables, population density, have included choropleth colors, dot density patterns, and even (inappropriately) graduated circles. None of these symbols are prototypical, nor are the data they represent, and therefore it is not surprising that confusion exists.

Visual Variables

Visual or spatial variables are the components of graphics--the distinct aspects or characteristics of a graphical object that can be varied independently. Systems of visual variables also describe the aspects of a symbol that can be used to represent information. Geographers often cite Bertin's (1983) variables: size (area), value, texture (pattern), color, orientation, and shape, along with the two planar dimensions (length and width). Garmiz (1990) proposes a similar list that includes internal structure, or patterns within cartographic symbols. Insight into the dimensionality of a cartographic symbol can be obtained by considering which visual variables are available for that symbol. As shown in the Table, each visual variable is prototypical for certain geometric dimensions. For example, "width" is possessed only by

This visual variable *. . .*	*. . . is* prototypical *for these dimensions . . .*	*. . . but has a* non-prototypical *status in these dimensions*
color/value	0, 1, 2	text
internal structure	2	0,1, text
length and width	2	0, 1
orientation	2, text	0
pattern	1, 2	0
position	0, 1, 2, text	*n/a*
shape	2, text	0,1
size/area	2	0,1, text

Table. Visual Variables as Indicators of Dimension

areas; it is a prototypical characteristic only of areal symbols. As suggested above, a highway symbol with internal structure is not prototypical because it possesses width. "Line" symbols that have width are not prototypical; a true line symbol cannot have internal dimensions. One can also refer to the Table for assistance in selecting appropriate cartographic methods once one has established the dimensionality of a set of data. For example, when mapping prototypical linear data, the appropriate visual variables are color, position, and pattern. Since position is usually fixed, one must vary either the color or the pattern of lines, without employing internal structure or other non-prototypical variables.

Benefits of the Cognitive Model

The cognitive geometry model proposed here is applicable only to two-dimensional paper maps. A more complete cognitive model will be the subject of future investigation. It will include symbols and variables for geometric dimensions zero through three, text, and sound, temporal, and tactual cartography. The main benefit of the cognitive approach to mapmaking is that maps created with reference to the cognitive model will be more accurate representations of their source data. The right mapping technique can be chosen with greater certainty when one uses a geometric scheme of much greater precision than the topologic data model and its cousins. Cognitive geometry need not replace cartographic topology in GISs; the cognitive approach can work whatever the method of data storage. No matter whether it is printed on paper or embedded in a word processor, a thesaurus still helps one select the best word. Similarly, while cartography would certainly benefit from software that used the cognitive model to propose appropriate mapping techniques, one can apply the concepts of cognitive geometry in any circumstances. Use of the new model will assist cartographers in determining the true dimensional character of their data, and will also spur the development of new techniques and symbols appropriate to specific types of data. Cognitive geometry will lead to better cartography, both more defined and more flexible.

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