A CASE STUDY OF SIMULTANEOUS GENERALISATION

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Abstract. Cartographic generalisation aims at simplifying the representation of data to suit the scale and purpose of a map. This paper deals with an algorithm that implements the whole graphic generalisation process (roughly defined as the operators simplification, smoothing, exaggeration and displacement) called *simultaneous graphic generalisation*. This method is based on constraints, i.e. requirements that must be fulfilled in the generalisation process. The constraints strive to make the map readable while preserving the characteristics of the data, which implies that all constraints cannot be completely satisfied. This paper shortly describes the theory of simultaneous graphic generalisation and presents a case study of the method. Visual assessment of the result of the case study indicates that the method gives good graphic generalisation results.

Keywords: cartographic generalisation, graphic generalisation, optimisation, least-squares method

1 Introduction

Cartographic generalisation aims at simplifying the representation of cartographic data to suit the scale and purpose of the map. Much research during recent decades has been devoted to automation of the generalisation process (Müller et al., 1995). The process has been divided into about ten operators, and several algorithms have been proposed to implement these operators. More than one operator is required to solve most generalisation problems and, accordingly, the optimal sequence of operators must be determined (see e.g. Nickerson, 1988; Mackaness, 1994; Regnauld et al., 1999; Ruas, 1999). One problem of the sequential approach is that the operators have different goals. When an algorithm is applied to solve a conflict, the algorithm may create other conflicts that have to be solved by subsequent algorithms. Another common problem with the sequential approach is that objects are (normally) treated in isolation. That is, if the geometry of an object is generalised there might be new (spatial) conflicts with neighbouring objects that must be identified and solved. To circumvent the problems of the sequential approach, algorithms should implement several operators in a single step, and they should also model the relationship between objects. This strategy has been suggested by several authors (Ware and Jones, 1998; Sarjakoski and Kilpeläinen, 1999; Sester, 2000), but no solution has been presented for implementation of the complete generalisation process in a single step. The development of such an algorithm is difficult, in particular for model generalisation (generalisation due to changes in the conceptual model).

A good map must satisfy several requirements: the map must be visually legible and the cartographic objects must be a good representation of reality. These requirements can act as constraints in the generalisation process (Beard, 1991; Brassel and Weibel, 1988). Recently several generalisation methods have been developed based on analytical constraints (Ruas and Plazanet, 1996; Burghardt and Meier, 1997; Harrie, 1999; Højholt, 2000; Ruas, 2000; Sester, 2000; Harrie and Sarjakoski, 2001). Constraints are interesting both for model generalisation and in graphic generalisation (moving and/or distorting objects to make the data visually legible; roughly the same as the operators: simplification, smoothing, exaggeration and displacement). However, this paper concentrates on constraints for graphic generalisation.

Harrie and Sarjakoski (2001) have described an optimisation method of graphic generalisation called *simultaneous graphic generalisation*. The method is based on a number of analytical constraints,

where some are defined for single objects and some for groups of objects. Ideally, all these constraints should be fulfilled. However, the constraints are contradictory, some of them strive to change the data to make the map readable, while others strive to maintain the characteristics of the data. The solution will be a compromise between the constraints and, depending on the weighting of the constraints, different solutions are obtained (strategies for weight-setting are described in Harrie, 2001).

The aim of this paper is to describe the theory of simultaneous graphic generalisation and to present a case study of the method. The next section presents a general introduction to constraints. Then follows a brief description of simultaneous graphic generalisation and section 4 describes the case study. The paper is ended with concluding remarks.

2 Constraints

Several requirements must be fulfilled in the generalisation process. A possible framework for automatic generalisation is to formulate these requirements as constraints and let them control the process (Beard, 1991). The major difference between rules and constraints is that rules state *what is to be done* and constraints state *what results should be obtained*. In this paper, three main categories of constraints are presented: *legibility*, *characteristic* and *position* (a modification of the typologies given in Ruas and Plazanet, 1996 and Weibel and Dutton, 1998).

Legibility constraints

The visual representation of cartographic objects is important. The data must not contain any spatial conflict, objects (and features within objects) must be large enough and not too detailed, and the chosen symbolisation must conform to graphic limits.
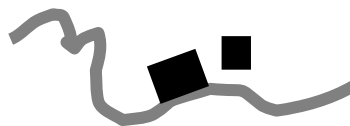

Figure 1: Two legibility constraints are violated. Some of the bends in the road object are too narrow, and there is a spatial conflict between one of the building objects and the road object.

Characteristic constraints

It is essential that the characteristics of single objects, as well as of groups of objects, are maintained in the generalisation process. Several constraints have been proposed for single objects, such as preservation of area and angularity. Defining constraints on groups of objects is more difficult, and often requires the preservation of object patterns, which is particularly difficult if objects are removed. Examples of constraints on groups of objects are: alignment of objects (Regnauld, 1996), mean distance between objects (Anders and Sester, 2000) and size distribution (Ruas, 2000).
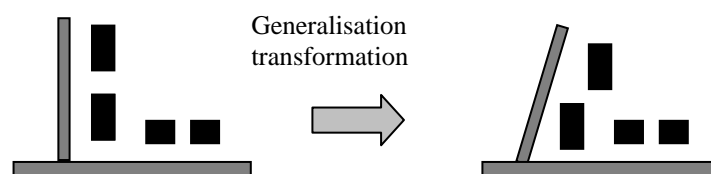

Generalisation transformation

Figure 2: Two characteristic constraints are violated in the generalisation transformation. The angle between the road objects at the junction has changed, and the large building objects are no longer aligned.

Position constraints

Position constraints are concerned with the movement of objects in the generalisation process. There are two types of position constraints: *absolute* and *relative*. Absolute position constraints state that

objects should not move in relation to the geodetic reference system, while relative position constraints dictate that distances between objects, as well as topological relationships, must be maintained.

3 Simultaneous graphic generalisation

Simultaneous graphic generalisation aims at computing the optimal solution according to a set of analytical constraints. The constraints are expressed as linear equations of point movements (of all points that make up the objects included in the generalisation process). This method implies that the constraints will constitute an equation system in which the unknowns are the point movements. Solving this equation system, which is performed by the least-squares method, gives the "ideal" point movements required to distort and move objects in the graphic generalisation transformation.

This section starts by presenting the constraints used in simultaneous graphic generalisation. Then, the general form of these constraints is stated, as well as the equation system formed by the constraints. Finally, the least-squares method is briefly described, a method that is used for solving the equation system.

The description of simultaneous graphic generalisation here is brief. Definitions of the analytical form of the constraints, rules for when the constraints are set up, computational details, etc., are given by Harrie and Sarjakoski (2001) and weight-setting strategies are discussed in Harrie (2001).

Constraints in simultaneous graphic generalisation

Simultaneous graphic generalisation has ten constraint types (as presented in Harrie and Sarjakoski, 2001) but more constraint types can be added. These constraint types are described below using the typology presented above. One should be aware that the displacement and exaggeration constraints may have different goals, depending on the context in which they are used. If there is a spatial conflict, the displacement constraints strive to increase the distance between the objects, otherwise the constraints strive to maintain the distance between the objects. Exaggeration constraints can be used to increase the size of an object and to maintain the shape.

Legibility constraints
- Displacement: Spatial conflicts are not allowed. These constraints are set up when the distance between objects is shorter than a predefined minimum distance.
- Simplification: Line and area objects should not contain more points than necessary to represent their characteristics. The simplification constraints force an unnecessary point to lie on the straight line between the two neighbouring points; the points that are unnecessary is determined by the area-based algorithm in Visvalingam and Whyatt (1993). The reason for not simply removing the unnecessary points is to control the spatial relationships to other objects; however, the unnecessary points are removed during a post-processing in our application, using the Douglas-Peucker algorithm (Douglas and Peucker, 1973).
- Smoothing: Line and area objects should not be too angular.
- Exaggeration: Objects, and features within objects, should be large enough to be clearly visible.

Characteristic constraints
- Curvature and Segment length: The characteristics of line and area objects must be maintained.
- Stiffness: The internal geometry of some objects must be invariant.
- Crossing: The angle between line objects in junctions must not change.
- Exaggeration: The shape of some objects must be maintained.

Position constraints
- Movement: Points should not move (absolute).
- Movement direction: Points on a line should not move in any direction across the line (absolute).
- Displacement: The distance between close objects that are not in spatial conflict, should not change (relative). These constraints are set up when the distance between objects is greater than the predefined minimum distance, but shorter than 1.5 times the minimum distance.

General analytical form of the constraints

One key issue is to find analytical expressions for the constraints. In simultaneous graphic generalisation the number of points is invariant, which enables the formulation of the constraints on point movements. For the sake of computational simplicity, we restrict ourselves to linear equations; that is, all the constraints are of the form:

$$const_{x1} \cdot \Delta x_1 + const_{y1} \cdot \Delta y_1 + ... \; + const_{xn} \cdot \Delta x_n + const_{yn} \cdot \Delta y_n = const_{obs} \tag{1}$$

where

$\Delta x_i , \Delta y_i$	are point movements,
$const_{xx}$	are constant values, and
n	is the total number of points.

All the constraints together constitute an equation system in which the point movements are the unknowns. In matrix form this can be written as:

$$\mathbf{Ax} = \mathbf{l} + \mathbf{v} \tag{2}$$

where

A	is the design matrix,
x	is a vector containing the unknown point movements,
l	is the observation vector (containing the right-hand side of Equation (1)), and
v	is the residual vector.

The residual vector has to be introduced since the Equation system (2) is over-determined. The method guarantees that either a movement or a simplification constraint is set up for each *x*- and *y*-coordinate. That is, there are always at least as many constraints as unknowns, and in realistic applications there are about twice as many constraints as unknowns.

Least-squares method

The "best solution" of Equation system (2) is the one that agrees as far as possible with the constraints, i.e. we face a minimisation problem of a function of the residual vector. To solve the equation system the least-squares method is used, which minimises a weighted l_2 norm:

$$\mathbf{v}^\mathrm{T} \mathbf{P} \, \mathbf{v} \tag{3}$$

where

P	is the weighting matrix, and
$\mathbf{v}^\mathrm{T}$	is the residual vector transposed.

The least-squares solution is given by:

$$(\mathbf{A}^\mathrm{T} \mathbf{PA}) \cdot \mathbf{x} = \mathbf{A}^\mathrm{T} \mathbf{P} \, \mathbf{l} \tag{4}$$

A normal-sized graphical generalisation application contains thousands of points, and Equation system (4) will contain twice as many unknowns as points. To solve these large equation systems we use the conjugate gradient method (as proposed by Sarjakoski and Kilpeläinen, 1999), which is a computationally efficient method for this kind of application. To solve the graphic generalisation problem in Figure 3, containing almost 1000 points, took a few seconds on a PC with Pentium II, 266 MHz processor. The processing time is, however, highly dependent on the application and the input parameters.

The solution of Equation system (4) is dependent on the weights in matrix **P**. These weights could be set by apriori knowledge about the relative importance of the constraints, by a trial-and-error strategy, by using machine learning, or by a post-processing (cf. Harrie, 2001).

4 A case study using the constraint violation strategy for weight-setting

In this case study, a cartographic data set on the scale of 1:10,000 was generalised to the scale of roughly 1 : 50,000 (see Figure 3). The data were generalised as follows. Firstly, a model generalisation step was performed interactively using built-in functionalities in the map production software LAMPS2 (Laser-Scan, 1999) and some of our own routines. In this case study, the model generalisation step was only a preparatory step for graphic generalisation, and therefore, the model generalisation was not evaluated. Secondly, graphic generalisation was performed by a C++ implementation of simultaneous graphic generalisation which communicates with LAMPS2 via ASCII files (see Harrie and Sarjakoski, 2001 for details). The quality assessment of the graphic generalisation was performed both visually and quantitatively.

Model generalisation

The original cartographic data set consisted of the object types: *building-residence*, *building-other*, *major road*, *minor road*, *dirt road*, *field* and *power line*. The new data set had almost the same object types. The major difference was that the building objects in this data set were of type *building-point* (point objects) or *building-area* (area objects). The following rules were applied in the model generalisation step (see Figures 3a and 3b).

Building objects
- *Building-residence* and *building-other* objects larger than 250 m^2 were represented as *building-area* objects with the same geometry as the original objects.
- *Building-residence* objects smaller than 250 m^2 were represented as *building-point* objects, where the direction of the symbol corresponded to the longest side of the original *building-residence* object.
- *Buildings-other* objects smaller than 250 m^2 were not represented.

Road objects
- *Minor road* objects shorter than 100 metres and not leading to a *building-residence* object were omitted.
- *Dirt road* objects were not represented.

Field objects
- Small islands were removed.

(a)

Model
generalisation

Figure 3: Case study of simultaneous graphic generalisation.

(a) the original map, (b) the model-generalised map, (c) and (d) the model- and graphic-generalised map.

In (a) the *building-residence* objects are represented in black and other building objects in grey. In Figures (b), (c) and (d) *building-area* objects are represented in grey and *building-point* objects are represented by black squares.

The original cartographic data was provided by the National Land Survey of Sweden. Printed by permission 507-98-4091.

Graphic generalisation

(b)

(c)

(d)

Graphic generalisation

This step was performed by simultaneous graphic generalisation, where the weights of the constraints were set by the constraint violation strategy; this weight-setting strategy is based on that the user defines apriori values of how much each constraint type is allowed to be violated (see Harrie, 2001).

In the new data set, six object types were used. Objects of these types should behave differently in the graphic generalisation process. *Building-area* objects are regarded as being rigid objects. In this case study, the rigid property was implemented by only allowing small deviations from the exaggeration constraints. The reason to use exaggeration constraints, rather than stiffness constraints, was that the *building-area* objects should be enlarged at the same time as the shape should be maintained. For *building-point* objects only the position of the object was of interest; hence, only displacement and movement constraints were used. *Major* and *minor road* objects should be plastic in the graphic generalisation process, and the objects should be simplified and smoothed. Therefore, simplification, smoothing, movement, curvature, segment length and movement direction constraints were used as individual constraints for these objects, and displacement and crossing constraints for relationships to other objects. The *allowed violation values* were generally smaller for the *major road* objects than for *minor road* objects. *Power line*, and *field* objects should also be plastic. However, these objects represent angular entities; therefore, smoothing constraints were not set up for these objects.

Assessment of the result of the graphic generalisation step

The results of the simultaneous graphic generalisation were assessed both visually and quantitatively. The visual assessment was performed by studying Figures 3b and 3c and details of the quantitative assessment are given in Harrie (2001).

In the model-generalised map (Figure 3b) there were some conflicts that had to be solved by graphic generalisation. These conflicts were mainly spatial conflicts between *road* and *building* objects, *field* objects that were too detailed, and *building-area* objects that were too small in relation to *building-point* objects. However, there was no need for major simplification or smoothing of the *road* objects

By studying Figure 3c we can see that most conflicts were solved. For example, there are some spatial conflicts and a size conflict (too small *building-area* object) in the top left circle in Figure 3b. As shown in Figure 3d, the simultaneous graphic generalisation process was able to solve these conflicts. There were basically just two circumstances under which the spatial conflicts could not be solved. The first case was related to a lack of free map space and the second was dependent on the rules for setting up displacement constraints; the latter is illustrated here by an example. In the bottom right circle in Figure 3b, the *building-area* object was too small and was too close to the *road* object just left of the *building-area* object. Between the *building-area* object and the left part of the *minor road* object a displacement constraint was set up. However, when the *building-area* object was enlarged and moved away from the *road* object a new spatial conflict was introduced to the same *road* object below the *building-area* object. Due to the rules for setting up displacement constraints, no constraint was set up between the *building-area* object and that part of the *road* object. See Harrie and Sarjakoski (2001) for definitions of the rules for setting up displacement constraints and a discussion about their applicability.

5 Concluding remarks

Graphic generalisation is required both in map production and in real-time navigation systems. Today, most automatic routines for graphic generalisation are based on sequentially applying algorithms for the graphic generalisation operators, where the objects are treated in isolation. During recent years, a number of generalisation methods based on a compromise between constraints have been developed. This paper deals with one of these methods: simultaneous graphic generalisation. This method solves the graphic generalisation in a single step, where relationships between objects are also modelled. This paper presents a case study of simultaneous graphic generalisation. Visual assessment of the results of this case study indicates that this method gives good graphic generalisation results.

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