The Cartographic Generalization of Hydrographic Feature Based on the Fractal Geometry

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1. Abstract

This paper discusses the method for measuring the fractional dimension of hydrographic feature. The fractional dimension of coastlines and rivers on the series scale topographic map is measured by using the method .the change regularity of the fractional dimension of hydrographic feature is found as fellow:

(1) The smaller scale of the map is, the smaller the fractional dimension of hydrographic feature is.
(2) The smaller scale of the map is, the smaller the self-similarity is.

Coastlines and rivers on the map are generalized by using the change regularity and fractal geometry. The result of the cartographic generalization is satisfied.

2. Introduction

Cartographic generalization is always an important problem in cartography. It should be based on quantitative objective standard, so that it is avoided subjective factors. Cartographers are ever studying the problem. Today with the wide use of computer, development of GIS, necessary of cartographic generalization, the problem is especially prominent. Since French math scientist Mandelbrot first found Fractal Geometry theory in 1945, it has been used in many fields. Now, the introduction of fractal dimension made it possible to describe the complex, self-resembled phenomenon. It provides a new way for quantitative standard in cartographic generalization.

3. Fractal phenomenon of hydrographic feature in cartography

Coastline and river in the map have fractal characteristics. When we use different characteristic measurements to describe these features, we can find powerful function relation between step length and section. That is to say that coastline and river have fractal characteristics. Of course, step length should have a range. That range is also called no scalar area. The length of curve can be described,

$$L_n=R_n*N_n$$

R_n is characteristic length of the N_{th} change, N_n is the number of sides.

And we know,

$$N_n=c/R_n^d$$

So

$$L_n = C/(R_n)^{d-1} \quad ,$$

$$D = 1 + \ln(L_{n+1}/L_n)/\ln(R_n/R_{n+1})$$

L_n is the total length of all the n times changes, C is a constant.

To river, besides main stream and tributaries that have fractal dimension as coastline, from other angle, rivers-net seems a tree composed of tributaries and main stream. Main stream is regarded as main trunk. Tributaries are regarded as branch trunk. Trunk is divided into tributaries. Tributaries also are divided again. It is self-resembled layer structure.

4. Determining the fractal dimension of hydrographic feature

4.1 The method of determining the fractal dimension of hydrographic feature

After researching and analyzing, we think step-measure method is suited to determining hydrographic feature fractal dimension. The method is introduced as fellow,

(1) Measuring cartographic curve L by different step R_i and recording curve length $L(r_i)$,(i=1,2,3,4,5……n),

(2) In log-log coordinate system , linear regression method is used to imitate these dot pair(log Ri , log L(ri)), and we can get,

$$\log L = A + B * \log R$$

(3) The fractal dimension "D" of cartographic line can be determined by

$$D = 1 - B.$$

(4) Relf-resembled degree can be got by linear regression coefficient,

$$R = \Sigma\,(\ln R_i * \ln L(r_i)) / \sqrt{(\Sigma(\ln R_i)^2 * \Sigma(\ln L(r_i))^2)}$$

4.2 Measuring the fractal dimension of coastlines and rivers on series scale maps

In order to find out the change regular pattern of the geographic feature in different scales, we have measured three coastlines and three river systems by selecting step length 1 km, 2 km, 3km, 4 km, 5km to measure them. The measuring data and analysis result can be found in the following tables.

Table one: Measuring data of fractal dimension of rivers and calculate results

Group Number	Scale	Number measured by step length(N)					Dimension (D)	Self-resembled degree
		1km	2km	3km	4km	5km		
I	1:50000	61	23	11	8	6	1.465114	0.983362
	1:100000	60	23	11	8	6	1.455317	0.976721.
	1:200000	50	22	11	7	5	1.449001	0.964399
	1:500000	35	17	9	7	5	1.212939	0.935356
	1:1000000	27	12	7	5	4	1.203444	0.840258
II	1:50000	56	22	11	6	5	1.557087	0.981706
	1:100000	55	21	11	6	5	1.538795	0.974703
	1:200000	54	20	10	6	5	1.528261	0.961908
	1:500000	24	11	7	5	3	1.235492	0.932851
	1:1000000	17	8	5	4	2	1.231242	0.832704
III	1:50000	56	21	11	6	4	1.639525	0.980232
	1:100000	55	21	11	6	4	1.628846	0.972580
	1:200000	50	20	11	6	4	1.564372	0.960583
	1:500000	19	8	5	3	3	1.209897	0.933272
	1:1000000	18	8	5	3	3	1.177851	0.839877

Table two: Measuring data of fractal dimension of coastlines and calculating results

Group Number	Scale	N measured by pace length					Dimension	Self-resembled degree
		1km	2km	3km	4km	5km		
I	1:50000	33	14	8	6	5	1.197498	0.987089
	1:100000	33	14	8	6	5	1.197498	0.981450
	1:200000	33	14	8	6	5	1.197498	0.971085
	1:500000	32	14	8	6	5	1.179259	0.936781
	1:1000000	32	14	8	6	5	1.179259	0.936781
II	1:100000	70	30	20	12	8	1.314221	0.979794
	1:200000	69	30	20	12	8	1.305693	0.969075
	1:500000	65	30	19	11	8	1.297873	0.932898
	1:1000000	62	29	18	11	8	1.269042	0.839275
III	1:100000	64	28	16	11	8	1.292405	0.980180
	1:200000	62	27	16	11	8	1.267637	0.969992
	1:1000000	62	25	15	11	8	1.260681	0.934419

From the table data, we can draw some conclusions.

(1) When we use the same step to measure the same river or coastline in different scales, the smaller the scale is, the smaller the fractal dimension is. Cartographic line changes from complexion to simplicity.

(2) In different scales, the smaller the scale is, the smaller the self-resembled degree is.

(3) "D", the fractal dimensions of rivers have much change between 1:200000 and 1: 500000. The result shows that 1:50000 topographic maps are generalized too much in our country.

5. The application of fractal geometry in the cartographic generalization of hydrographic feature

Fractal dimension is the parameter that reflects complexion and stuff capability of space in geographic feature. If we find change regular about fractal dimension of geographic feature in serial scales, we can utilize the regular to instruct cartographic generalization.

5.1 The application of fractal geometry in the generalization of coastline

The length of curve is always regarded as the selected standard in the generalization. With the observing scale and map scale changing, the number of curve is continuously changing. If we know that the number of curve should be chose and the number of curve should be deleted in the scale of map, it is should become easy to get the key to cartographic generalization.

The number of curve is decreasing with the length of curve is increasing. So we can get that,

$$n_A=N*e^{-\alpha X_A}\ldots\ldots\ldots\ldots..(1)$$

N is the total number of curve, X_A is the criterion of selection, α is the coefficient of curve. n_A is number of curves selected on the new map.

α is regarded as the parameter for judging curve in coastline. On the other hand, characteristic of α is just the same as that of "D". The more complex coastline is, the bigger α is, the bigger value of "D" is. So we can draw an conclusion that the coefficient of curves (α) is the same as the fractal dimension of "D".

In general,

$$0<\alpha<1$$

And,

$$1<D<2$$

So

$$\alpha = D-1 \quad \text{or} \quad D = \alpha + 1$$

Therefore

$$n_A = Ne^{-(D-1)X_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Number of curve of coastline in every scale can be determined by the equation. The calculating and analysis result can be found in the table three.

Table three: Selecting number of coastline curves determined by fractal cartographic generation math model

Group number	Scale	Dimension (D)	Curve select Criterion :X(mm)	Actual selected number	Curve number determined by model	Absolute error
I	1:50000	1.197498		39		
	1:100000	1.197498	12	33	31	-2
	1:200000	1.197498	12	24	24	0
	1:500000	1.179259	12	17	20	+3
	1:1000000	1.179259	12	12	16	+4
II	1:50000	1.314221		39		
	1:200000	1.305693	12	28	27	-1
	1:500000	1.297873	12	20	19	-1
	1:1000000	1.269042	12	13	14	+1
III	1:100000	1.292405		62		
	1:200000	1.267637	12	47	45	-2
	1:500000	1.260681	12	31	33	+2

The difference between the number of curve calculated by math model and actual selection is very small in table three. Error is in allowable extent. So we can use fractal cartographic math model to determine selected the number of the curves.

5.2 The application of fractal geometry in the generalization of river

River is one of the major features in map. Determining selected number of rivers is the key problem in cartographic generalization. The selected number of rivers is determined according to the following equation,

$$n_A = N * e^{-\alpha L_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

n_A is the selection number, N is the number of rivers on the material map ; α is parameter , which changes according to the complexion of rivers; L_A is selection criterion.

α is regarded as a parameter to judging the complexion of river, whose characteristic is as same as that of "D".

In general,

$$0<\alpha<1$$

$$1<D<2$$

So

$$\alpha = D-1$$

Therefore

$$n_A = N * e^{(1-D) L_A} \ldots\ldots\ldots\ldots (4)$$

D is the fractal dimension of rivers on new-complied map.

Table four: Selecting number of rivers determined by fractal cartographic generation math model

Group Number	Scale	Dimension (D)	River selected criterion L_A (km)	Actual selected number	Rivers number determined by model	Absolute error
I	1:50000	1.465114	0.35	31		
	1:100000	1.455317	0.7	24	23	1
	1:200000	1.449001	1.4	12	12	0
	1:500000	1.212939	3.5	5	6	-1
	1:1000000	1.203444	7	1	1	0
II	1:50000	1.557087	0.3	40		
	1:100000	1.538795	0.6	30	29	1
	1:200000	1.528261	1.2	15	15	0
	1:500000	1.235492	3	6	7	-1
	1:1000000	1.231242	6	2	2	0
III	1:50000	1.639525	0.25	24		
	1:100000	1.628846	0.5	19	18	1
	1:200000	1.564372	1.0	11	10	1
	1:500000	1.209897	2.5	5	6	-1
	1:1000000	1.177851	5	1	2	-1

Selected criterion of river is decided based on the fractal dimension .In general, the bigger the value of the fractal dimension is, the lower criterion is. But selected criterion of river should be changed into actual size according to scale in calculation. The theoretical selection number of rivers computed by fractal cartographic generalization math model is similar to actual number by measuring in table four. So it is feasible, scientific and reasonable to use fractal dimension to direct rivers cartographic generalization.

6. Fractal dimension relation between adjacent scales in map

It is not enough to know only fractal dimension change regular under different scales in cartographic generalization. For example, generalized rivers on 1:100000 by using 1:50000 map. If according to (4) equation, we should know the fractal dimension of rivers on 1:100000 map, but we only know that on 1:50000 map. So we have to find out fractal dimension relation between adjacent scales.

Supposing, M_1 is the scale in material map, L_1 is the length of curves, D_1 is fractal dimension. M_2 is the scale in compiling map, L_2 is the length of curves, D_2 is fractal dimension.

$L_1=c/(r_1)^{D_1-1}$

$$L_2=c/(r_2)^{D_2-1}$$

According to measure regular,

$$r_1/r_2=M_1/M_2$$

$$L_2=L_1* (r_1)^{D_1-1}/(r_2)^{D_2-1}$$

$$r_2= r_1* M_2/ M_1$$

So,

$$L_2=L_1*(r_1)^{D_1-1}/(r_2)^{D_2-1}* (M_1/M_2)^{D_2-1}= L_1*r_1^{(D_1-D_2)} *(M_1/M_2)^{D_2-1}$$

According to Beckett Equation,

$$L_2=L_1*(M_2/M_1)^{-0.017}$$

So,

$$(M1/M2)^{0.017} = r1^{(D_1-D_2)} * (M1/M2)^{D_2-1}$$

$$(M1/M2)^{1.017-D_2} = r1^{(D_1-D_2)}$$

Change it by log power,

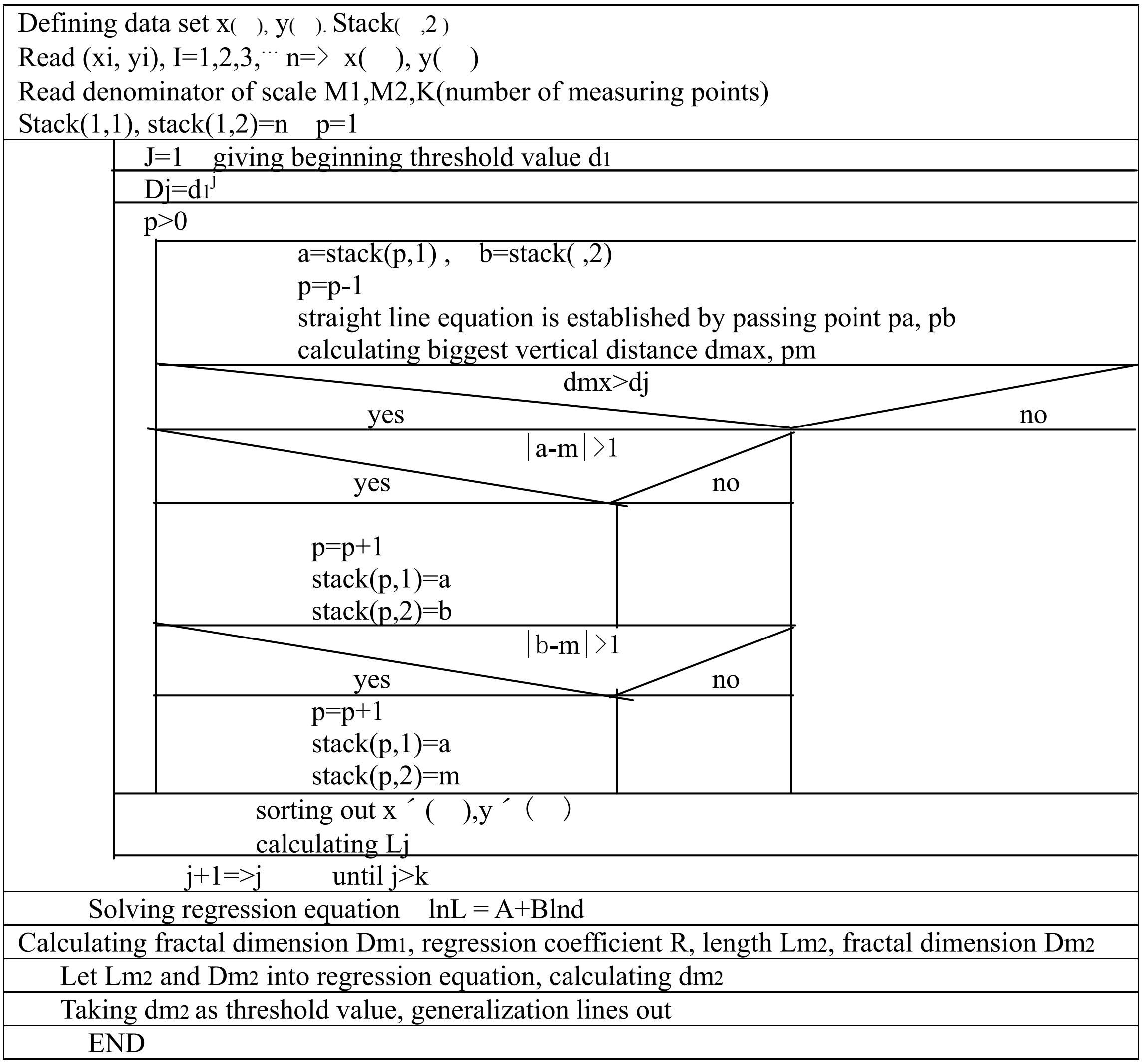
$$1.017\ \ln(M1/M2) - D_2 * \ln(M1/M2) = D_1 * \ln r_1 - D_2 * \ln r_1$$

$$D_2 = D_1 * \ln r_1 - 1.017\ \ln(M1/M2) / \ \ln r_1 - \ln(M1/M2)$$

Threshold value r_1 is determined before changing. Only one value D_2 can be determined by r_1. At the same time threshold value r_2 in new compiled map is determined by D_2 and L_2 , thus realizing threshold value determined in self-adaptability.

7. The application of fractal geometry in computer cartographic generalization of curve

The method to determine threshold value, we may do the curve generalization with algorithm in computer cartography application. Such as Douglas algorithm, supposing r is step length, d is tolerance value, r and d both have character of fractal geometry. So d is regarded as “r”. The following algorithm is Douglas algorithm improved by fractal geometry theory.



The result of the cartographic generalization by fractal geometry is satisfied.