A Model of Point Cluster Generalization with Spatial Distribution Features Recognized and Measured

Yi Lu*   Jinghai Du*   Jingsheng Zhai**

*Dept. of Marine Surveying & Mapping,
Dalian Naval Academy,
1, Xiao Long St., Dalian, China (116018)
E_mail: lulutian@public.tpt.tj.cn

**Tianjin Institute of Hydrographic Surveying & Charting,
40, You Yi Road, Tianjin, China (300061)

Abstract

Following the line generalization, point cluster generalization becomes the second focus in cartographic generalization field, for many objects distribute in random fashion, such as islands，lakes and houses etc. Point cluster generalization contains three aspects: quantity selection, structure selection, and single object simplification. Obviously, structure problem is the key and difficulty among the three ones. Preserving the structure information in spatial distribution is the basis of point cluster generalization.

In this paper, based on the Delaunay triangulation and Voronoi diagram model, it focuses on the discussion of spatial distribution properties by recognition and measurement. Four characteristic parameters are defined for distribution property description: distribution density of three dimensions, distribution range of two dimensions, distribution axes of one dimension, distribution center of zero dimension. With the aid of Delaunay triangulation and Voronoi diagram, representation and calculation models of above-mentioned parameters are established.

1. Delaunay Triangulation and Voronoi Diagram

The Delaunay triangulation and Voronoi diagram are important data structures in computational geometry, and have been used in many applications. This paper presents a research on how to apply these data structures to support our problem solving.
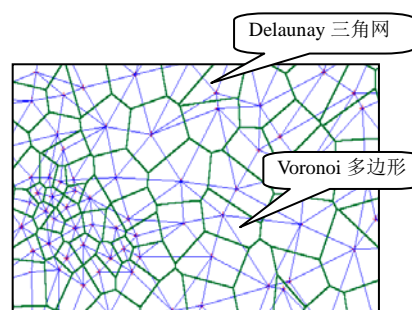

Fig.1 Delaunay Triangulation and Voronoi Diagram

Basic Properties of the Delaunay Triangulation

The Delaunay triangulation is a triangulation of a set of points V with the empty circle property, that is that the circumcircle of any of its triangles does not contain any point of the given set. The Delaunay triangulation is unique and locally equiangular, hence , it maximizes the minimum angle of its triangles compared to all other triangles (Figure 1). The Voronoi diagram is the dual graph of Delaunay triangulation, also called Thiessen polygons, subdivides the space into a set of convex tiles whose boundaries are the perpendicular bisectors between adjacent data points (Figure 1).

Constrained Delaunay Triangulation(CTD)

The pure Delaunay triangulation is notable for providing a direct solution to the all-nearest neighbors problem for a set of point sites, in that each vertex is connected to its nearest neighbors. CDT is a modification of a conventional Delaunay triangulation, in which edges, belonging to vector-defined map objects, are enforced as edges within the triangulation. The constraining edges act as barriers to visibility of nearby vertices that are on the opposite sides of such edges. In practice, for polygonal and linear objects, their nearest neighboring objects are usually directly connected by triangle edges. Exceptions occur particularly where there are constraining edges that are relatively long. The neighboring objects to a specified object can be found easily by a search procedure that starts with the triangles externally connected to the object. The triangulation is a complete representation of a specified region, in that every point of the triangulated region belongs either to the interior of a single triangle or to an edge or a vertex of one or more triangles. The precision of coordinates can be arbitrarily high and the fact that space is partitioned into triangles makes measurement of areas of parts of objects and of parts of free space relatively straightforward. Topological relationships between points, lines, and areas can be explicitly represented within a triangulation data structure.
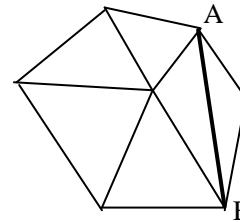

Fig.2 Constrained Delaunay Triangulation

One of the most important properties of the CDT for map generalization is that of the rich proximity relations. Proximity relations may be easily understandable in case of objects that are connected to each other. But disjoint objects can also have proximity relations as long as any part of them are connected by a Delaunay triangle edge. The Delaunay neighbors of an object are not necessarily its nearest neighbors in geometric sense. Distant points may be accepted as neighbors in sparse areas, whereas relatively close objects may not be accepted as neighbors in dense areas if they occur behind other closer objects.

2. Recognition and Measurement

Measurement of Distribution Scope

In general, distribution scope of point cluster is an uncertain problem, for the polygon represents the scope is not one and only, such as tangent rectangle, circularity, convex and etc. Among them convex is the most popular and reasonable in the geometry field. However, it is not suitable for any point cluster. For example, when the point cluster has obvious concave parts, the convex can't express the fact exactly. Just as figure 3 describes. Obviously, figure (b) is more reasonable than (a), in the same way, (c) is more reasonable than (b). We may conclude that one of the important rules of express the scope of point cluster is that it must be measure up to the habit of human vision.
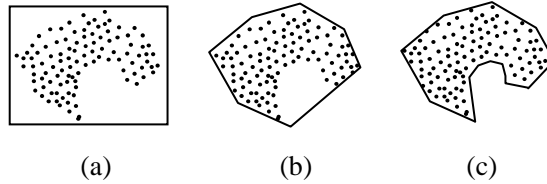

(a)　　　　　(b)　　　　　(c)

Fig.3　Scope Description of Point Cluster Distribution

According to the vision principle, given a set of point, we always regard those outside adjacent points as the boundary. Hence, to the convex, if the distance of two adjacent points is farther, the other point around it should replace one of them. In this way, this paper proposes a method named "nibble" which approach to the exact scope by nibbling the outside triangles.

Theorem 1: Delaunay triangulation $Tri = \{tri_1, tri_2, \cdots, tri_m\} = \{l_1, l_2, \cdots, l_n\}$, where tri_i is triangle, l_i is the edge of triangle, n>m，if $l_k \in tri_i$ and $l_k \in tri_j$（$i \neq j$）, then l_k must be the inner edge of triangulation, otherwise it must be one part of the boundary.

Prove：

Suppose S is the plane which Tri lies on, boundary of Tri is B , then B separates S into S_1 and S_2 . S_1 expresses the inner part of S , including boundary itself, S_2 the outside part.

According to theorem 1, $tri_i \in S_1$, $tri_j \in S_1$.

Condition 1: $l_k \in tri_i$ and $l_k \in tri_j$ where $(i \neq j)$

Condition 2: $l_k \in B$

Use apagoge, suppose condition 1 and condition 2 are all true, well then according to the properties of Delaunay triangulation, tri_i and tri_j must be lie on the opposite side of l_k . We can suppose $tri_i \in S_1$, so according to condition 2, it must be $tri_j \in S_2$.

This conclude conflicts the condition $tri_i \in S_1$ and $tri_j \in S_1$ in Theorem 1. Hence the postulated conditions can not be true.

<div align="right">Prove over</div>

Theorem 2: $Tri_i = (l_1, l_2, l_3)$, and $l_1 \in B$, if after deleting l_1, tri_i will not exit any longer, then $l_2 \in B$ and $l_3 \in B$.

The process of "nibble" method is as follows:
1) Construct the Delaunay triangulation of point cluster.
2) Based on Theorem 1, detect the triangle composing the boundary, meanwhile give threshold L, if length of edge (which composing the boundary) is longer than L, then delete this triangle.
3) Repeat step 2, until the length of each edge of boundary is less than L.

See figure 4, it is obvious that result (b) is more reasonable than (a). Furthermore, different results can be gained by using different threshold values, so the continuous-scale display may become real.

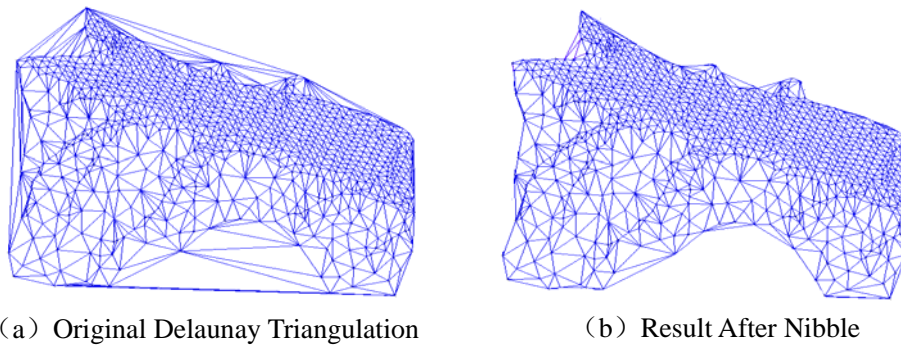（a）Original Delaunay Triangulation   （b）Result After Nibble

Fig.4   Measurement of Distribution Scope

Measurement of Density and Recognition of Center

In this paper, the distribution density is represented by Voronoi cell size and visualized as gray image. So the density can be changed into the area that can be defined and measured easily, and the center can be gained by extracting the peak value of gray image. We may know where is denser and where should be simplified firstly. But one thing must be pointed out here, every point should be regarded as nonobjective, without strong or weak, they divide the space on the equal principle. It is the basis and abides by the Voronoi principle.

Definition 2: Voronoi diagram $V = \{p_1, p_2, \cdots, p_n\} = \{V(p_1), V(p_2), \cdots, V(p_n)\}$, where p_i is point, $V(p_i)$ is the Voronoi cell, the density is

$$dens(p_i) = 1/A(V(p_i))$$ ………………………………. ……………..(3)

where, $A(V(p_i))$ is the area of $V(p_i)$。

The larger $A(V(p_i))$, the sparser points, and the smaller $dens(p_i)$. Therefore, we can conclude, distribution center will retain as long as the distribution density does not change more.

4

Extraction of Distribution Axes

In this paper, axes is extracted by using the boundary polygon which describes the distribution scope. At first, construct the constrained Delaunay triangulation of boundary polygon (figure 5). Secondly, line the edge center of every triangle orderly. The line can be seen as the axes of point cluster (figure 5).
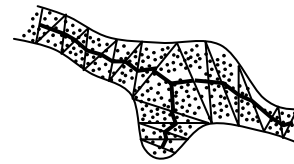

Fig.5 Extraction Distribution
Axes of Point Cluster

3. Point cluster generalization

It is not difficult to find that all the parameters (scope, density, center, axes) mentioned above are not irrelative. If the scope does not change much after generalization, the axes will retain accordingly. If the distribution density keeps the original state, the center will not be far away from the former place. So the constrained condition can be from four to two. Generalization may be focused on the scope and density features.

Generalization in a Dynamic way

A generalization model of point cluster is provided in this paper on the basis of Voronoi diagram establishment in a dynamic way. With the condition of rules, the densest place must be the first to be simplified. According to the formulation 3, the point (P) locates at the densest place should be detected and deleted first. The cell corresponding to this point should be partitioned by its adjacent points (F_1, F_2, F_3, F_4), and in order to avoiding the continuous deletion in the same area, these points should be marked. Figure 6 expresses this process. Repeat the step until the result satisfies the standard. Times is determined by the scale span.

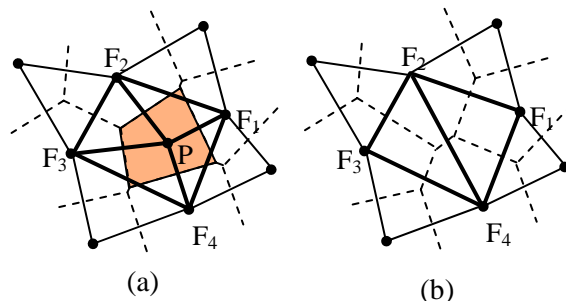(a)   (b)

Fig.6   Update   of   Delaunay
Triangulation and Voronoi Diagram

It is necessary to say that when the triangulation is the constrained Delaunay triangulation, the influence of deleting one point does not only act on the adjacent points(F_1, F_2, F_3, F_4), but would be affect the second layer(such as point S, figure 7), which are the adjacent points of F_1, F_4 (figure 7).

Generalization model

Based on the measurement and recognition mentioned above, referring the rules of point cluster generalization,
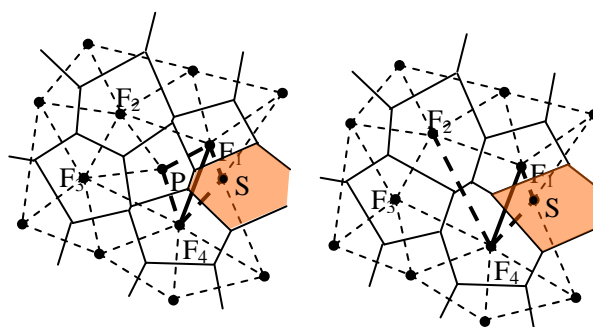

Fig.7   Update   of   Constrained   Delaunay
Triangulation and Voronoi Diagram

the generalization model is as follows:
1) Construct the Delaunay triangulation and Voronoi diagram model of point cluster P, and distinguish those points outside according to the third definition proposed in the paper.
2) According to the first and second theorem, by use of nibbling triangulation, gain the scope of P.
3) Simplified the outline, keep the characteristic point A、B、C、D、E (Figure 8).
4) Calculate the density of P_i and order them.
5) Delete P_i at the densest place, and re-construct part around it.
6) Sign the adjacent points of P_i for fear of deleting the same area continually.
7) Repeat step 5 and 6 until all the points have been signed. Up to now, one resample process has been completed.
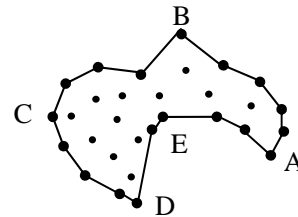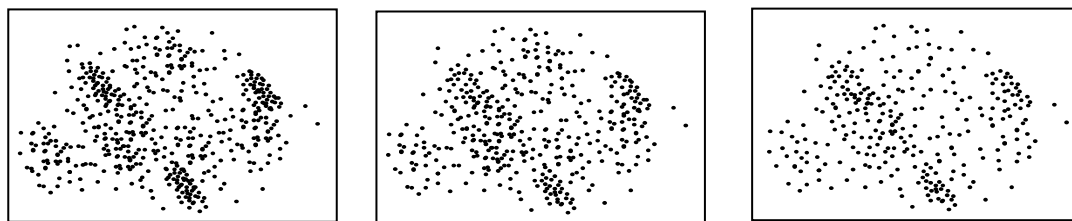8) Times of resample process is up to the span of scale.

Fig.8 Extraction of Outline

Figure 9 gives the example, (a) is origin, (b), (c) are the results after 2, 4 times extraction.

(a) Point Cluster   (b) Result After Resample 2 times   (c) Result After Resample 4 times

Fig. 9   Experiment of Point Cluster Generalization

4. Concludes

Point cluster object contains much structured information in spatial distribution. Based on the Delaunay triangulation and Voronoi diagram, this paper defines four characteristic parameters for structure information. And by establishing the Voronoi diagram in a dynamic way, a method of point cluster simplification is provided. In a word, the generalization model preserves the spatial distribution properties of point cluster very well. And all the methods mentioned above has been proved to be feasible and sound.

5. References:

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