A KNOWLEDGE BASED APPROACH
FOR THE GENERALIZATION OF LINEAR FEATURES

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ABSTRACT

Generalization of linear features is considered to be among the most important generalization operations. This is due to the fact, that the majority of map features are represented as lines or polygons, which are bounded by lines. Usually the results of a generalization transformation are evaluated visually, mostly based on aesthetic criteria and less on the qualitative/quantitative assessment of the result. Generalization of digital data is an analytic procedure and thus specific criteria must be developed for the evaluation of the results. This paper elaborates a methodology enabling the assessment of linear features generalization integrity, through the utilization of specific tools. Measures are used to describe quantitatively the effects of generalization and assessment tools determine whether specific constraints are satisfied. New measures for the quantification of shape change due to generalization, based on the parametric description of line shape, are used along with measures for the assessment of the horizontal position error. Specific conditions are checked, utilizing the measures and the interpretation of the post-generalization situation. The application of alternative generalization schemas and assessment tools to the original data, result to the selection of the optimal solution. In this way structural knowledge is related to a specific generalization solution and procedural knowledge is acquired.

1. GENERALIZATION AUTOMATION: CARTOGRAPHIC KNOWLEDGE AND QUALITY

Generalization results - in order to be acceptable - must comply with certain quality constraints. This can be accomplished with the introduction of cartographic knowledge into the generalization process. Three types of cartographic knowledge: geometrical, structural and procedural have been identified (Armstrong 1991). Geometrical knowledge refers to the actual geometry or topology. Structural knowledge arises from the generating process of an object or results from computations performed on the cartographic data and is used to guide generalization. Procedural knowledge leads to the selection of appropriate generalization operators in a given map context. The application of cartographic knowledge in the generalization process ensures the satisfaction of the quality constraints. The development of a framework for the evaluation of the post-generalization quality is akin to cartographic knowledge as measures, which formally describe and record cartographic knowledge, can be used for the identification of quality constraints.

Positional accuracy can be maintained through the application of procedural knowledge and it is a generalization quality element. The average distance between the original and the generalized line is a measure used for the assessment of generalization results. Therefore:

- When computed in the framework of an "acceptable" generalization solution i.e. a manually generalized line, it records knowledge (knowledge acquisition).

- When computed for the assessment of a generalization solution i.e. the result of a simplification algorithm, it provides a measure of the accuracy of generalization results (quality evaluation).

As the majority of map features are lines or polygons, which are bounded by lines, cartographers give special attention to the automatic generalization of linear features. Positional accuracy of linear features can be analyzed to horizontal position accuracy and shape fidelity (European Committee for Standardization, 1996). Shape is a major factor and one of the components of positional accuracy. As such, it must be preserved through generalization. Moreover, it is related to the way the cartographer handles linear features. This is the reason why structural knowledge, which describes shape, plays such an important role in line generalization. Structural knowledge describes formally and objectively, the way that the cartographer conceptualizes and characterizes line shape. Line segmentation into homogeneous parts along with objective shape description, result to the allocation of linear segments into groups of similar shape. This process enables the cartographer to make a considerable step towards automation of generalization: to apply different generalization transformations to each group. A methodology for the parametric description of line shapes and the segmentation of lines into homogeneous parts (Skopeliti and Tsoulos, 1999) has been proposed, along with measures for the quantification of shape change due to generalization (Skopeliti and Tsoulos, 2000; 2001).

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The objective evaluation of generalization results with measures which describe shape and horizontal position change, contribute to the research on the positional accuracy aspect of cartographic generalization, in two ways:

- **Development of generalization assessment tools**: Objective measures, which describe positional accuracy, can lead to the formulation of tools for the evaluation of automatic generalization results. These tools reflect the constraints pertaining to each cartographic entity and are derived from established cartographic principles. For example, a basic constraint for the generalization of linear features is the preservation of their original character. The measures, which quantify the change of shape due to generalization, can be used to judge whether the results fulfill this constraint. If a combination of algorithms and tolerance values is applied to each group of lines, then the optimal solution - with respect to the features' characteristics - can be assessed based on the satisfaction of the relevant constraints.

- **Knowledge acquisition**: Recent research aims at the correlation of acceptable generalization solutions (combination of operators, algorithms, tolerance values) with line shape, which is described by a group of parameters. In particular, Weibel et al. (1995) apply a machine learning technique to derive prototype rules, which relate the tolerance values of the Lang simplification algorithm to groups of lines with different complexity. Line shape (structure recognition) is described with a group of parameters (cluster analysis identifies several ranked classes per attribute). Procedural knowledge is provided by human judgment, from the selection of the Lang algorithm tolerance values, in order to match manual generalizations. In addition, Lagrange et al. (2000) use neural networks to relate the tolerance values of the Gauss smoothing algorithm with groups of lines with different complexity. Line shape is described by a group of parameters and procedural knowledge is provided by the interactive identification of a range of parameters for the smoothing algorithm, utilizing - as reference - manually generalized features. In this paper, a different approach is proposed. Measures, which describe the positional accuracy, are computed for manually generalized data or cartographically acceptable generalization results, which serve as exemplars. These measures are also computed for the generalization results, which are produced by automated procedures. Through these measures, "exemplars" and "automated generalization" results can be compared quantitatively and related objectively without human interaction. On the other hand, the methodology for shape and structure recognition is applied to original data (Skopeliti and Tsoulos, 1999). Thus structural knowledge is described formally and lines are clustered into similar shape groups. Utilizing machine-learning techniques, structural knowledge can be related to the automatic generalization solution, which is closer to the exemplars and procedural knowledge can be acquired.

This paper elaborates the identification of tools for knowledge acquisition on generalization results, which will be used for the development of a comprehensive knowledge base, which will support automated line generalization.

2. SHAPE DESCRIPTION AND STRUCTURE RECOGNITION

According to the methodology for the parametric description of linear features shape (Skopeliti and Tsoulos, 1999), three parameters: the average magnitude angularity plot, the error variance and the ratio of length to the base line length, are utilized. They were selected from a broad set of parameters, utilizing Principal Components Analysis (PCA). The selection of a surrogate variable for each factor is based on the information resulting from the statistical analysis and cartographic expertise. The measures selected, exhibit low correlation between them and therefore cluster analysis can be applied for the classification of line segments into similar shape groups. Further utilization of the parametric description of linear features shape, leads to formation of measures for the assessment of generalization effects on line shape.

In order to achieve a successful classification for linear entities, lines should be homogeneous along their entire length. The development of a segmentation methodology is therefore a prerequisite. A considerable number of researchers have already focused on linear features segmentation such as: Plazanet et al. (1995) using man-made features, Wang and Muller (1998) and recently Dutton (1999) using coastlines. The methodology for linear features partitioning into homogeneous segments (Skopeliti and Tsoulos 1999) addresses natural linear features and is based on line shape assessment utilizing the fractal dimension and the parametric description of lines shape.

3. MEASURES FOR POSITIONAL ACCURACY

Positional accuracy is influenced by a number of generalization operators such as simplification, smoothing etc. The assessment of the generalization results, calls for the quantitative description of horizontal position and shape.

3.1 Horizontal Position

Researchers have identified a number of measures for the assessment of the positional deviation between the original and the generalized line. The measures used in this work are: a. the average Euclidean distance from the

original to the generalized line or from the generalized to the original line b. the Hausdorff distance (Abbas et al., 1995) and c. the ratio of the area between the original and the generalized line to the length of the original line (McMaster, 1987). The average change of the horizontal position for a group of lines is equal to the average of the change values calculated for the individual line segments.

3.2   Line shape

The shape of a line in its original or generalized form can be described with the use of the above-mentioned group of parameters. In addition, shape change of a linear feature with respect to its original form, can be assessed through the parametric description of line shape. **In cluster analysis, the distance between two lines in the parameters' space implies similarity. The distance between the original and the generalized line implies shape change**. This is a quantitative assessment of shape change due to generalization. The average value of the shape change for the lines, which make up a group, represents the average line shape change for the particular group.

A qualitative assessment of line shape change on the individual line segment level is achieved, through the examination of the cluster analysis results for the generalized lines in comparison to the cluster analysis results for the original data. When non-hierarchical cluster analysis is applied, using the centers of the original lines groups, the results describe the generalized lines similarity to the original lines. For example the allocation of a line into a group of smoother lines, indicates that the simplification algorithms result to less complex lines. When hierarchical cluster analysis is applied, the results show the similarity between the generalized lines.

4.   CONSTRAINTS, MEASURES AND ASSESSMENT TOOLS

When assessing the generalization quality at the line level, the following constraints can be identified (Weibel 1996): metric (aspects of perceptibility), topologic (avoid self-intersection), structural or semantic (shape preservation) and gestalt (preserve original line character). *Assessment tools* can determine whether constraints are satisfied. *Measures* are needed to describe quantitatively the generalization effects. Specific *conditions* utilizing the results of the measures and the post-generalization situation can *be interpreted*.

Regarding the positional accuracy aspect of the generalization results the following *constraints* can be considered: maintenance of shapes variety, maintenance of global complexity, preservation of the original lines characteristics, degree of simplification suitable for the new map scale, minimization of shape distortion and minimization of horizontal position error. These constraints belong to the metric, structural and gestalt categories. From the study of the positional accuracy of generalization results (Skopeliti and Tsoulos, 2000; 2001) the following measures are proposed: the average change of shape for all or each group of lines, the results of the generalized lines clustering and the average horizontal position error for all or each group of lines. These measures are elaborated in the following paragraphs, in order to lead to the identification of the assessment tools.

The values of the **average change of shape** for a group of lines or for all lines must ensure a degree of data simplification, which is justified by map scale and map purpose and at the same time, shape fidelity.

In order to describe the **clustering results of the generalized lines**, the following measures can be used:

In hierarchical classification **the number of groups** indicates the variety of shapes, which exist in the generalized data and the ability to distinguish between them. In the non-hierarchical classification the number of groups indicates the degree of generalization. The number of line classification groups diminishes as the line shape is simplified.

The synthesis of the generalized lines groups: When assessing the results of the generalized lines clustering, a number of cases are observed: membership of the initial groups is retained, membership of the initial groups changes, new complexity groups are created etc. The synthesis of the generalized line groups is described quantitatively through percentages for every case in each clustering. These percentages express the number of lines belonging to each case to the number of lines in the groups of the original lines.

In the **hierarchical classification** results, the following cases can be identified:

  I. Segments are classified as in the original classification: This means that the algorithm preserves the characteristics of the lines in the group, all segments undergo the same degree of simplification and they constitute a newly formed group.

 II. Segments are classified with others, which were initially classified into a group of less complex lines: The algorithm does not treat all segments in the same way and their shape has been distorted. As a result, they are similar to other lines of different original shape and new groups are formed. This is an undesirable phenomenon, which should be avoided.

III. Segments are classified with others, which, in the initial clustering, were classified into a group of more complex lines. This case commensurate with the previous one.

In the **non-hierarchical classification** results, the following cases can be identified:

I. Segments are classified into the same group as in the initial classification: The change in line shape is not enough to lead to the degradation of their complexity.

II. Segments are classified into the group of less complex lines: Line shape is moderately simplified.

III. Segments are classified into the group of the least complex lines: Line shape is sensibly simplified.

Although all cases in the non - hierarchical classification are acceptable, the new map scale against the generalization degree should be considered. **The average horizontal position error** should not influence the horizontal position accuracy in relation to the new map scale.

Constraint	Measure	Conditions	Interpretation
Maintenance of shapes variety	The number of groups in the hierarchical classification	1. The number of groups in the hierarchical classification is equal to the number of original lines classification groups	The variety of shapes of the original data is retained
		2. The number of groups in the hierarchical classification is less than the number of the original lines classification groups	The variety of shapes of the original data is reduced
Maintenance of global complexity	The number of groups in the non-hierarchical classification	3. The number of groups in the non-hierarchical classification is equal to the original lines classification groups	Moderate complexity reduction
		4. The number of groups in the non-hierarchical classification is less than the number of the original lines classification groups	Sensible complexity reduction
Preservation of the original lines characteristics	Percentages of the cases in the hierarchical classification	5. Case I in hierarchical classification, exhibits considerable percentage. Cases II and III exhibit a low percentage	Preservation of original lines characteristics
Degree of simplification suitable for the new map scale	Percentages of the cases in the non - hierarchical classification	6. Case I in non-hierarchical classification exhibits considerable percentage	Lines have been moderately simplified
		7. Case II in non-hierarchical classification exhibits considerable percentage	Lines have been moderately simplified
		8. Case III in the non-hierarchical classification exhibits considerable percentage	Lines have been sensibly simplified
Minimization of shape distortion	Average shape change	9. Average shape change values are small	Shape distortion is moderate
Minimization of horizontal position error	Average horizontal error	10. Average horizontal position error values are less that the legibility threshold at the map scale.	Horizontal position error is acceptable

Table 1. Assessment tools are formed on the basis of measures and conditions, which lead to the interpretation of the generalization results.

Utilizing the above-mentioned measures, assessment tools can be formed for the generalization constraints (Table 1). For example, the constraint for the maintenance of shapes variety is checked utilizing the number of groups in the hierarchical classification measure. If condition #1 is "TRUE", an interpretation of the generalization results that "the variety of shapes of the original data is retained" is acquired. On the contrary, if condition #2 is "TRUE", an interpretation of the generalization results that "the variety of shapes of the original data is reduced " is acquired.

Three kinds of results can be anticipated: a condition is satisfied, a condition is not satisfied and a condition is partially satisfied. The importance of the constraints fulfillment, depends on the scale change and the purpose of

the resulting map. The calculation of the measures and the estimation of the assessment tools should be performed for each group of lines and map scale.

The implementation of alternative generalization schemas on the original data and the application of the assessment tools will result to the interpretation of the post-generalization situation and the selection of the optimal solution.

5. CASE STUDY

The above-described methodology, for the assessment of the consequences of generalization to the horizontal position and the shape of linear features, has been applied to a number of simplification algorithms. A solution, which satisfies generalization constraints at the individual segment level for the 1: 500 000 map scale, will be identified. Simplification is preceded by structure recognition and shape description. The data set used in the experiment, is the coastline of the Greek islands Ithaca and Lefkada, digitized from a 1:100 000 scale map. The specific coastline was selected due to its complicated configuration. The coastline is segmented into eighteen (18) segments (Figure 1), which are classified into four (4) groups according to their shape: "very smooth" (VSM - group 1), "smooth" (SM- group 2), "sinuous" (SIN- group 3 and "very sinuous" (VSIN- group 4). Simplification of linear segments is applied through the following algorithms:

♦ Global routines: Douglas Peucker (DP),

♦ Local processing routines: Reuman – Witkam (RW), Lang (LG) and Euclidean distance (ED)

♦ Routines taking into account structure: ESRI Bendsimplify (BS) (Wang and Muller 1998)

The tolerance value, which is used with the algorithms, is selected to be equal to the legibility threshold (0.25 mm) at the new map scale.
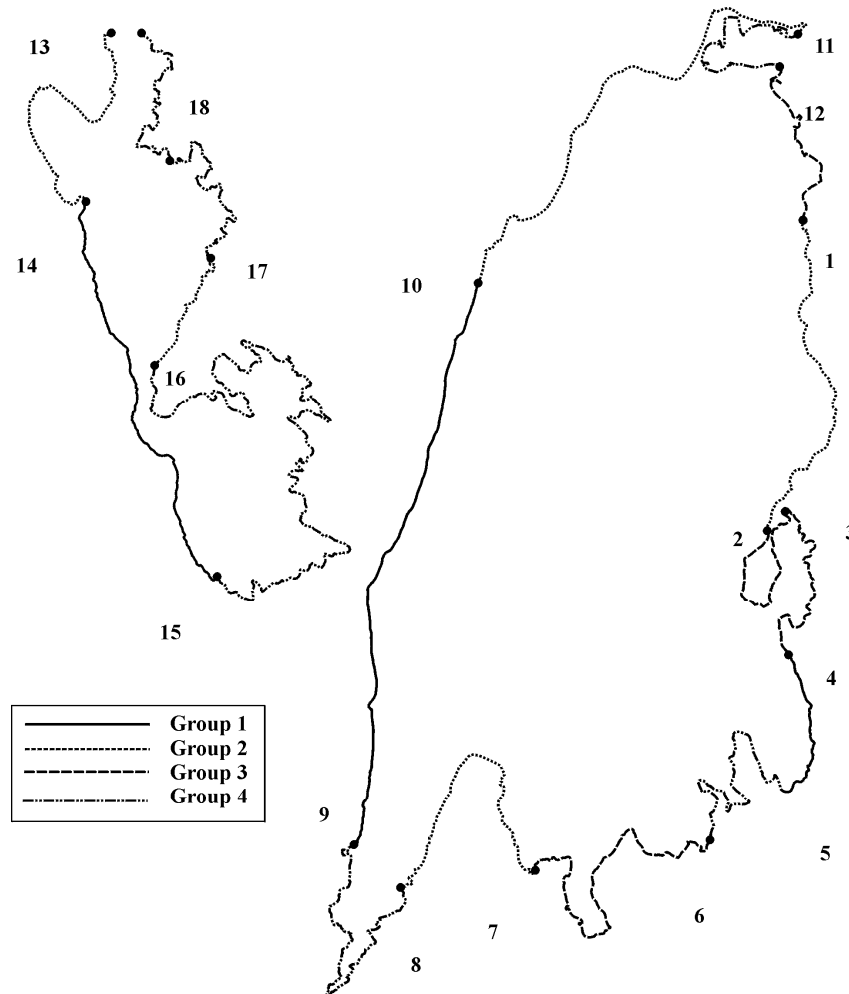

Figure 1. Segmentation and classification results of the coastline (Ithaca and Lefkada islands).

5.1 Performance of assessment tools

Horizontal Position accuracy is assessed in relation to the legibility threshold in the new map scale. For the 1: 500 000 scale, the legibility threshold is equal to 125 meters. From the average horizontal position error

values (Table 2), it is apparent that all values are smaller than the legibility threshold, except from the Hausdorff distance values for Reuman – Witkam algorithm. Therefore, condition #10 is satisfied for all algorithms except for the Reuman – Witkam.

Algorithms	Average Euclidean Distance from the original to the	Average Euclidean Distance from the generalized to the	Average Hausdorff Distance	Number of Groups in Hierarchical	Number of Groups in Non -Hierarchical
DP	33.78	29.03	115.23	4	4
RW	44.14	32.07	165.57	3	4
ED	11.25	6.53	49.32	4	4
LG	12.47	9.33	61.39	3	4
BS	11.91	3.19	65.99	4	3
Original				4	4

Table 2. Horizontal Position accuracy measures and number of groups in clustering.

In Table 3, **the number of groups** where the simplified lines are classified into, as a result of hierarchical and non-hierarchical classification, is presented. The Douglas-Peucker, the Euclidean distance and the Bendsimplify algorithms satisfy condition #1, whereas Lang and Reuman Witkam algorithms satisfy condition # 2. The Douglas-Peucker, the Reuman-Witkam, the Lang and the Euclidean distance algorithms, satisfy condition #3, whereas the Bendsimplify algorithm satisfies condition #4.

The synthesis of the generalized lines groups is evaluated through percentages of Cases I, II and III for each clustering (Tables 3 and 4), algorithm and group of lines, through the application of conditions # 5, 6, 7 and 8. A condition on classification results is considered as fully satisfied, when the percentage value is equal to 100.

Algorithm			Hierarchical					Non - Hierarchical				
Code	Group		DP	RW	ED	LG	BS	DP	RW	ED	LG	BS
4	VSM	1	1	1	1	1	1	1	1	1	1	1
9		1	1	1	1	1	1	1	1	1	1	1
14		1	2	1	1	1	1	1	1	1	1	1
1	SM	2	2	1	2	2	2	1	1	2	2	2
7		2	2	1	2	2	1	1	1	2	2	2
10		2	2	1	2	2	1	1	1	1	1	2
13		2	2	2	2	2	2	2	2	2	2	2
16		2	2	1	2	2	2	1	1	2	2	2
2	SIN	3	3	2	2	3	2	2	2	3	3	3
3		3	3	3	3	4	3	3	3	3	3	3
6		3	3	2	3	3	3	2	2	3	3	3
8		3	4	2	3	4	3	3	2	3	3	3
12		3	3	2	3	3	3	3	2	3	3	3
5	VSIN	4	4	3	3	4	3	3	3	4	4	4
11		4	4	3	3	4	3	3	3	3	3	4
15		4	4	3	3	4	3	3	3	3	3	4
17		4	4	3	3	4	3	4	4	4	4	4
18		4	4	3	3	4	3	3	3	3	3	4
Nu. of Groups		4	4	3	3	4	3	4	4	4	4	4

Table 3. Hierarchical and non-hierarchical classification results of the lines simplified by the Douglas-Peucker, Reuman -Witkam, Lang, Euclidean Distance and Bendsimplify algorithms.

Conditions are established for each group:

Very Smooth: Bendsimplify, Lang, Euclidean distance and Reuman-Witkam algorithms satisfy condition # 5, whereas all algorithms satisfy condition # 6.

Smooth: Lang, Euclidean distance and Douglas-Peucker satisfy condition #5. Bendsimplify satisfies condition #6, Lang and Euclidean Distance algorithms partly satisfy condition #6, Douglas-Peucker and Reuman-Witkam partly satisfy condition #7.

Sinuous: Douglas-Peucker and Euclidean distance partly satisfy condition #5. The Euclidean distance, Bendsimplify and Lang algorithms fully satisfy condition #6, Reuman-Witkam partly satisfies condition # 7.

Very Sinuous: Douglas-Peucker and Lang algorithms fully satisfy condition #5. The Bendsimplify algorithms fully satisfy condition # 6, and Douglas-Peucker and Reuman-Witkam algorithms partly satisfy condition # 7.

C	Alg	Very Smooth				Smooth				Sinuous				Very Sinuous			
		I (%)	II(%)	III(%)	A S.C.	I (%)	II(%)	III(%)	A S.C.	I (%)	II(%)	III(%)	A S.C.	I (%)	II(%)	III(%)	A S.C.
H	DP	67	0	33	6.67	100	0	0	6.27	80	0	20	5.63	100	0	0	5.86
	RW	100	0	0	6.86	0	100	0	7.89	0	80	20	9.46	0	100	0	9.75
	LG	100	0	0	1.75	100	0	0	3.33	60	0	40	2.79	100	0	0	3.98
	ED	100	0	0	1.48	100	0	0	2.41	80	20	0	2.25	0	100	0	2.99
	BS	100	0	0	0.17	60	40	0	0.18	80	20	0	0.28	0	100	0	0.17
NH	DP	100	0	0		20	80	0		60	40	0		20	80	0	
	RW	100	0	0		20	80	0		20	80	0		20	80	0	
	LG	100	0	0		80	20	0		100	0	0		40	60	0	
	ED	100	0	0		80	20	0		100	0	0		40	60	0	
	BS	100	0	0		100	0	0		100	0	0		100	0	0	

Table 4. Clustering results expressed in percentages for the Very Smooth, Smooth, Sinuous and Very Sinuous groups of lines (C: Clustering, Alg: Algorithm, H: Hierarchical, NH: Non - Hierarchical, A. S. C: Average Shape Change).

Regarding **the average shape change** (Table 4), the selection of a criterion for the condition satisfaction, is difficult since there is not a threshold value for this quantity. One option is to select the algorithm, which produces the minimum average shape change. In this case, the Bendsimplify algorithm satisfies condition #9.

The above described analysis of the assessment tools through the fulfillment of certain conditions, is summarized in table 5. An algorithm, which satisfies a considerable number of conditions, can be proposed for the generalization of each group of lines. For each algorithm and group of lines, the number of fully satisfied conditions is summed and divided by the total number of constraints. In this way a grade is assigned to each solution, which ranges from 0 through 1. Moreover, a verbal interpretation of the results is provided.

	Line group	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Number of criteria satisfied	Grade
DP	global	✓		✓							✓	3	0.5
	VSM						✓					4	0.7
	SM					✓						4	0.7
	SIN												0.0
	VSIN					✓						4	0.7
RW	global		✓	✓								2	0.3
	VSM					✓	✓					4	0.7
	SM												0.0
	SIN												0.0
	VSIN												0.0
LG	global		✓	✓							✓	3	0.5
	VSM					✓	✓					5	0.8
	SM					✓						4	0.7
	SIN						✓					4	0.7
	VSIN					✓						4	0.7
ED	global	✓		✓							✓	3	0.5
	VSM					✓	✓					5	0.8
	SM					✓						4	0.7
	SIN						✓					4	0.7
	VSIN												0.0
BS	global	✓			✓					✓	✓	4	0.7
	VSM					✓	✓					6	1.0
	SM						✓					5	0.8
	SIN						✓					5	0.8
	VSIN						✓					5	0.8

Table 5. For each algorithm and group of lines certain conditions are fully satisfied.

In this case study, the Bendsimplify algorithm is recommended for all groups, since it is characterized by the maximum grade. At the global level, the fulfillment of the conditions #1, #4, #9 and #10 show that the variety of shapes of the original data is retained, complexity is reduced sensibly, shape distortion is moderate and horizontal position error is acceptable. At the line group level, preservation of the original lines characteristics is achieved for the very smooth group (condition #5), whereas lines in all groups are considered as slightly simplified (condition #6).

6. CONCLUSIONS

The identification and adoption of assessment tools for generalization results constitute the basis for the development of a comprehensive conceptual framework for the automation of line generalization. Such tools have been identified, documented and evaluated in the framework of this study, accompanied by a well-defined methodology. This methodology has been applied in the framework of a pilot project with very promising results, stressing the need for a more in depth examination of the specific topic using a wider range of line types, algorithms and operators and a number of refined criteria. This process will result to the development of a knowledge base relating each type of line shape with the optimum algorithm and tolerance value and a set of rules to be used for the generalization of linear features at a particular scale or range of scales. The cartographic knowledge thus acquired can be further used in the development of the knowledge base in the framework of an expert system for the generalization of linear features.

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