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Fuzzy Urban Systems

Theory and Application to China

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Abstract:

This paper outlines a method for using the mathematics of fuzzy sets that is well suited to measure and characterize peri-urbanizing ("*desakota*") systems typical of China, Southeast Asia, and other areas experiencing rapid urbanization. Drawing on Kosko's "fuzzy hypercube", three distinct but interdependent measures are derived: (i) extent of urbanization, (ii) level of fuzziness, and (iii) degree of entropy. The feasibility of the proposed method is demonstrated using remote sensing data for Ningbo, China.

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Introduction

The *desakota* land use pattern that is characteristic of Southeast Asia, China, and other rapidly urbanizing regions is notable for its urban-rural ambiguity. Figure 1 re-presents the classic depiction of this peri-urbanization pattern as introduced by Terry McGee (1991). While much work has been done by urban geographers to understand and to document this phenomenon, less has been accomplished by way of systematic measurement of peri-urbanization. Indeed, the very nature of the phenomenon defies ready categorization and measurement and renders conventional measures obsolete.

figure 1 here: "McGee's desakota"

This paper addresses the measurement issue by drawing on the mathematical formulation of fuzzy sets. A fuzzy set is one for which the *degree of membership* for any element of the set may range from zero to one, and so is well suited to ambiguous or partial membership. In our context we are interested in *fuzzy urban sets*, the constituent parcels (or pixels) of which may exhibit varying degrees of inclusion. The fuzzy set formulation is a very natural one for *desakota* settings, and it is easy to envision, for example, how the degree of membership in the fuzzy urban set *U* may vary from one location to the next in figure 1. In contrast, conventional (or "crisp") sets, for which all elements are constrained to have full membership, seem ill-suited to *desakota* settings, as would be any crisp rendering of a peri-urbanizing territory into two discrete mutually exclusive and non-overlapping subsets, urban and rural.

This approach leads to three distinct yet integrated measures of urbanization for any given study area:

i. extent of **urbanization** -- the aggregate level of membership in the fuzzy set *U*;
ii. level of **fuzziness** -- the overall degree of ambiguity regarding membership in *U*;
iii. degree of **entropy** -- the extent to which membership in U is spatially diffused

Characterizing (remote sensing images of) urbanizing regions as *fuzzy urban sets* provides a single integrated perspective from which these three distinct measures are simultaneously defined. As we shall see, previous work by Bart Kosko (1992) provides a convenient and conceptually imaginative framework for this approach.

The next section of this paper indicates the focus of this work in the broader context of research on urbanization in China and elsewhere. Although the proposed method has relevance to other regions, China provides both the motivation for this work and the setting for this particular application. Section three explains the relevance of fuzzy set formulations for characterizing peri-urban (or fuzzy urban) systems, and goes on to show how Kosko's (1992) depiction of a fuzzy hypercube lends itself very naturally to the derivation of three relevant measures, all derived from a single conceptual framework. Section four demonstrates the feasibility of the proposed approach using 1987 and 1996 remote sensing images of Ningbo City, in China's Zhejiang Province. The concluding section discusses theoretical and practical issues that will need to be addressed if the proposed method is to be adapted to regular use.

Broader context of urbanization in China

Figure 2 depicts the modest (but useful) contribution of this paper in the broader context of research on urbanization in China. It is helpful in understanding both what the contribution is and what it is not. There are four elements identified in figure 2; each corresponding to a particular facet of urbanization as a field:

[1] China urbanization as a *desakota* geographic phenomenon
[2] remote sensing images of this phenomenon
[3] summary statistics about urbanization
[4] dynamic models of urbanization

Each of these is an area of study unto itself. Regarding the study of *desakota* urbanization as a geographic phenomenon [1], McGee's *desakota* descriptive model has been extended to the Chinese context by Zhou Yixing (1991), Yok-Shiu Lee (1991) and Guldin (1997). Similarly, Zhou Daming (1997) attempts to resolve the urban-rural dichotomy issue by outlining a descriptive model of "rural urbanization in China". The references above focus specifically on documenting and describing the *desakota* phenomenon in the Chinese context. There is also a voluminous literature addressing other aspects of urbanization in China but that is beyond the scope of our immediate focus.

figure 2 here: "Context of contribution"

Much work has also been done in the interpretation of remote sensing sensing images of urbanization, a focus that may be represented as [1 →2] in the context of figure 2. Remote sensing image data have been used to detect land use change in China by Yeh (2001) and by Li and Yeh (1998) applying and extending methods introduced by Howarth (1986) and by Mesev, Longley, Batty, and Xie (1995). Again, technical issues regarding the production of remote sensing image data are beyond the immediate scope of our inquiry, although such work is clearly of direct importance for our proposed method.

The method proposed in *this work takes a remote sensing image as its starting point* and develops summary statistics derived from those data. In the context of figure 2, therefore, our focus is on [2 → 3]. In this regard, our work has a similar focus to Anthony Yeh's (2001) recent application of entropy measures derived from remote sensing data to analyze urban growth in the Pearl River. Our work differs from Yeh's primarily with regard to the fuzzy set representation and interpretation of data, and also with regard to our simultaneous derivation and presentation of three urbanization measures derived from a single conceptual framework. As this [2 → 3] segment is the immediate object of our inquiry we shall save a more in-depth discussion of this topic for the next section.

More often, in the context of urbanization, summary statistics are derived without reference to or mediation by remote sensing image data. In the context of figure 2 we represent such efforts to summarize urbanization levels as a direct [1 → 3] path, rather than the indirect [1 → 2] & [2 → 3]. Most typical of such efforts are jurisdiction-based approaches that seek to define urban or rural territories or population based on the jurisdictions in which they are located. In essence, this approach defines urban territory (or population) as territory (or population) found within urban jurisdictions. Although this circular reasoning is highly unsatisfactory from a theoretical perspective, it is the most common source of official statistics available on urbanization, and it reflects an administrative orientation that is pre-occupied with fiscal and governance issues. These are certainly important issues, but for our purposes a fixation on jurisdictional boundaries tends to obfuscate the nature of the underlying geographic phenomenon.

This is not only a problem in China. The United States Census[1], for example, also defines the urban-rural dichotomy in circular terms (our emphases below):

> "Urban" consists of territory, persons, and housing units in:
>
> - Places of 2,500 or more persons incorporated as cities, villages, boroughs (except in Alaska and New York), and towns (except in the six New England States, New York, and Wisconsin), but excluding the rural portions of "extended cities".
> - Census designated places of 2,500 or more persons.
> - Other territory, incorporated or unincorporated, included in urbanized areas.
>
> Territory, population, and housing units not classified as urban constitute "rural".

[1] The definition is found in U.S. Census Bureau, *Urban and Rural Definitions*, October 1995.

While that definition may be functional at the level of Census Bureau operations, it does not shed light on the geographical phenomenon we are addressing. Equally unsatisfactory for our purposes is the standard textbook definition: "To an urban economist, a geographical area is considered urban if it contains a large number of people in a relatively high population density" (Sullivan, 1990, p.6) In both instances, place is given *a priori*, usually in jurisdictional terms, whereas *our approach seeks to uncover urban places inductively and empirically*.

The challenge in China is no less vexing, as evidenced by Gregory Eliyu Guldin's (1992, p. 5) plaint:

> "Never mind then that Rong Ma … makes a convincing argument for the superiority of a sociological view of urban community as the key to defining towns or that Kam Wing Chan argues against simplistic [dichotomous] categories. Forget that Ma and others … have pointed out the anomolies of the "town" (*zhen*) classification and its relationship to the varieties of county, district (*qu*), *xiang* and village towns, so the the "town" as encountered in Chinese statistical tables is neither an unchanging category nor a sociologically accurate one. Forget too that Clifton Pannell …, Graham Johnson …, and I argue that there are far too many interconnections among all areas of Chinese society simply to bifurcate the society into "rural" and "urban" spheres. Forget all these arguments for social scientifically-based categories or understandings, for in the end all of these must yield to the statistical tables of the State Statistical Bureau and *its* categories."

Thus, in both the United States and in China, the official state statistics used to record levels of urbanization are fixated on exogenously determined place boundaries, and those places are then classified as either urban or rural based on varying criteria. While such jurisdictional boundaries are of course necessary for effective administration of territories, they are no substitute for (and, indeed, tend to obscure) a direct examination of the underlying geographic phenomena associated with peri-urbanization.

Summary or descriptive statistics of urbanization are not so much ends in themselves; rather, they provide important feedback for land use planners and other decision-makers who may seek to intervene in ways that might generate more favorable future outcomes. Li and Yeh's (1998) or Yeh and Li's (1999) studies of the Pearl River Delta region are representative of studies that generate summary statistics from remote sensing data to monitor land use change. This is a direct [2 → 3] link in terms of figure 2, with an implicit [3 → 1] feedback link to policies regulating the form and extent of urban development. Other works develop and apply dynamic models to complete the feedback loop more explicitly. Zhou and Ma (2000), for example, use summary data to support a descriptive model of economic restructuring and suburbanization in China. In the context of figure 2 their work is best represented by [4 → 1] supported by [3 → 4], as is the work by Zhai and Ikeda (2000). The latter adapt a mathematical ecology model with differential equations that are fed by summary data on urban land use density and other ecological indicators. Another interesting variant of dynamic model is the cellular automatum as applied to urbanization by Batty and Xie (1994) and as applied to the Chinese urbanization context by Li and Yeh (2000). Cellular automata are fed directly by raster data, thereby bypassing the need for summary statistics and so can be represented in figure 2 by [4 → 1] supported directly by [2 → 4].

The brief sketch above by no means substitutes for a comprehensive review of China urbanization studies. It is, nonetheless, useful for clarifying where our particular contribution fits in within the larger scheme of things. Specifically, we focus on the link between [2 → 3], producing a meaningful set of summary statistics derived from remote sensing data of peri-urbanizing regions. We argue that such remote sensing images may be interpreted as fuzzy set data that reflect an underlying process of fuzzy urbanization, and we turn now to a more detailed description of the method by which such summary data may be derived.

Fuzzy Urban Systems

Fuzzy urban sets

Consider the set *U* and all possible members of *U*, $x_i \in X$, drawn from some universal set *X*, where $i \in N = \{1, 2, \ldots, n\}$. In classical set theory, membership in *U* is unambigously or "crisply" defined so that any element x_i of the universal set *X* is either a full member of *U*, or not a member at all. This bimodal characteristic of crisp sets is reflected in a membership function for *U*, $m_U(\,.\,)$, that only admits two possible values, zero or one:

$$(1) \qquad u_i \equiv m_U(x_i) \in \{0,1\} \quad \forall i \in N$$

For many purposes, including our own, this classical bimodal set membership function is unnecessarily restrictive, and so this motivates the introduction of *fuzzy sets*. Zadeh (1965) accomplished this simply by extending the membership function to map from *X* to the entire unit interval, so that

$$(2) \qquad u_i \equiv m_U(x_i) \in [0,1] \quad \forall i \in N$$

where equation 1 is now a special limiting case of equation 2. Fuzzy sets allow a continuum of membership values while crisp sets allow only for the two most extreme possibilities: full membership or no membership at all.

Now, consider a remote sensing image such as the one shown in figure 3, where each pixel x_i of the image corresponds to a particular parcel (or plot, or chunk) of land, and where the set X corresponds to the image as a whole, or more precisely, to the union of all individual parcels x_i. For each parcel we may in principle assign a degree of membership in the *fuzzy urban set* U in accordance with the membership function described in equation 2. As will be explained below, the shades of gray in figure 3 represent the degree of membership thus assigned, with white representing full membership and black representing no membership, and with all shades of gray corresponding to the continuum of values between these two extremes. The resulting set U (and by implication, its rural complement R) is "fuzzy" in the sense that the classical *yes-no* membership dichotomy now dissolves into a question of degree or extent.

figure 3 here: "Remote sensing image"

In graphical form, the extension from crispy to fuzzy sets is represented in figure 4, where in this illustration the reference set X (from which all member pixels x_i are drawn) is a real line representation of a 1-D geographical space. Here, the urban-rural dichotomy is transformed into a fuzzier notion of urbanity, where the height of the membership function determines the extent to which a given location x_i is a member of the set of all urban locations. If we take $u_i = 0.5$ as the threshold or cutoff point[2], then a "crisp" categorization of the space in figure 4 would result in the two crisp urban sets (and by inference, the three crisp rural sets) shown.

figure 4 here: "1-D urban sets"

This adjustment is significant for several reasons. First, it enables us to introduce ambiguity of land use classification formally and precisely. Fuzziness does not imply imprecision; rather, it implies a more precise way of handling ambiguous land use patterns. Fuzziness is intrinsic to the underlying *desakota* phenomenon that is being described[3]. Second, it allows us to draw upon and apply work done by Zadeh (1965), Kosko (1992) and others who have contributed to fuzzy set theory, even though their work would appear at first glance to be

[2] That is, x_i is a member of the crisp urban set U whenever $u_i \equiv m(x_i) \geq 0.5$.

[3] Put another way, it is the logic of fuzzy sets, not the fuzzy logic of sets. (This distinction is not recognized in the title to Heikkila's "Fuzzy Logic of Accessibility" paper.)

quite far removed from issues of urbanization in China. For example, let the rural set R be defined as the complement of the urban set U, that is $R \equiv U^c$. In the case of crisp sets, the intersection of any set with its complement is null:

$$(3) \qquad U \cap U^c = U \cap R = \emptyset$$

For fuzzy sets, though, this result no longer applies, and this is potentially significant for how we organize land use data. As another example, Kosko (1992) shows how familiar terms such as entropy can be recast in terms of fuzzy set operations, and we use his result directly in the section that follows. Third, and of most direct import for this proposal, casting peri-urbanization patterns as fuzzy urban sets leads directly to the use of three fundamental dichotomies for characterizing urban-rural systems.

Three fundamental dichotomies of fuzzy urban systems

In a recent paper Heikkila (2000) applies Bart Kosko's (1992) notion of a *fuzzy power set* to develop three fundamental dichotomies in the context of accessibility[4]. We adapt those same dichotomies here as fundamental descriptors of fuzzy urban systems. The three dichotomies are best understood in the context of Kosko's (1992) graphical depiction of a fuzzy power set, which is depicted here in figure 5. Consider the universal reference set $X = \{x_1, x_2, \ldots x_n\}$, which in our case refers to the set of pixels in a remote sensing image, and so n is a very, very large number. The power set of X, denoted by 2^X, contains all crisp subsets of X. For example, in the three-dimensional case (n=3) which is depicted in figure 5,

$$(4) \qquad 2^X = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1,x_2\}, \{x_1,x_3\}, \{x_1,x_3\}, X\}$$

These elements of 2^X (which are themselves sets) constitute the vertices of an n-dimensional hypercube such as the one in figure 5. This is the fuzzy power set of X. Using this representation, following Kosko (1992), the fuzzy urban set U can be depicted as a point within a hypercube. Its location with respect to each vertex is specified by the corresponding membership value. For example, in figure 5 the coordinates of U are given as[5]:

$$(5a) \qquad u_1 = m_U(x_1)$$

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[4] In "Fuzzy Logic of Accessibility", geographical accessibility is cast as a special case of membership in fuzzy clubs, which in turn are extensions of Tieboutian municipalities. Please refer to Heikkila (2000) for a more extensive discussion.

[5] Here, for obvious reasons, our rendering of the hypercube is limited to three dimensions, which in turn corresponds to a remote sensing image of only three pixels. The reader is asked to bear in mind at all times that our discussion is geared to the n-dimensional case where n is large.

(5b) $u_2 = m_U(x_2)$

(5c) $u_3 = m_U(x_3)$

Expanding upon Heikkila (2000) and Kosko (1992), we characterize the fuzzy urban set *U* in terms of three fundamental dichotomies as summarized in table 1:

Table 1: Three fundamental dichotomies of fuzzy urban systems

Nature of the dichotomy	***Geographic interpretation***	***Fuzzy set interpretation***	***Geometric interpretation (figure 5)***
Urban-rural	Aggregate level of urbanization for the study area	Measure of the **cardinality** of the fuzzy set U	Distance from ∅
Fuzzy-crisp	Extent of *desakota* phenomenon	A measure of the **fuzziness** of the set U	Proximity to midpoint M
Entropy-order	Spatial diffusion of urbanization process	**Uniformity** of membership of the set U	Proximity to central diagonal

figure 5 here: "Fuzzy hypercube"

Urban-rural dichotomy. This is a measure of the aggregate level of urbanization for the sample area. As such, it corresponds most closely to conventional or official statistics measuring urbanization. A key difference, however, is that each parcel (or pixel) may in principle be partially urbanized. Thus, for example, a study area of one hundred parcels that is sixty percent urbanized may be composed of:

a) sixty urban parcels and forty rural parcels, or

b) one hundred parcels, *each of which* is sixty percent urban and forty percent rural, or

c) any convex combination of (a) and (b).

Conventional measures, such as those based on the Census definitions reported earlier, only allow for type (a) distinctions, where any given *place* is either urban or rural but not both.

As noted earlier, the vertex *X* in figure 5 corresponds to a system that is completely urbanized (each pixel, x_i, is a full member of the set *U*). At the other extreme, the vertex ∅ corresponds to a completely rural system, where each pixel is a full member of *R*, defined as the complement of *U*. In the context of figure 5, therefore, the urban-rural dichotomy is measured in terms of the distance of *U* from ∅. This measure is operationalized most simply by taking the mean value of U's base vectors, the resultant value of which is automatically normalized to the zero-one interval[6]:

(6) $M(U) \equiv \Sigma_i u_i / n$

Fuzzy-crisp dichotomy. This interpretation is rooted in the potentially ambiguous nature of membership and, as has been explained above, corresponds most directly to the fuzziness or ambiguity inherent in *desakota* land use patterns. The height of ambiguity (ie, maximum fuzziness) occurs at the midpoint M in figure 5 where each parcel of land has a 0.50 degree of membership in the urban set U and a 0.50 degree of membership in the rural set R. In contrast, each vertex of the fuzzy power set corresponds to one of the crisp subsets of *X*. Conventional methods for measuring urbanization restrict our possibilities to the vertexes alone, while the approach proposed here opens up the interior spaces as well, all of which retain some degree of fuzziness. Following Kosko (1992), the midpoint M in figure 5 corresponds to the point of maximum fuzziness of a set. In the context of urbanization, this corresponds to the ultimate *desakota* condition, where urban is rural and where rural is urban. Kosko operationalizes this measure of fuzziness using fuzzy set operators:

(7) $F = M[U \cap U^c] / M[U \cup U^c]$

where $M(U) = \Sigma_i \| u_i \|$ is a measure of cardinality for a set, and where the intersection and union of two fuzzy sets are given by the pairwise minima and maxima, respectively. Note that when U is crisp (corresponding to any of the vertices in figure 5), fuzziness drops to zero while *fuzziness equals one at the midpoint*.

Entropy-order dichotomy. In the context of urbanization studies entropy measures the dispersion of observed development patterns. For example Yeh (2001), in his study of urban sprawl in the Pearl River Delta region of Southeast China, uses the standard entropy measure derived from information theory (Theil, 1967; Thomas, 1981)

(8) $E = \Sigma_i p_i \log(1/p_i) / \log(n)$

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[6] Another alternative would be to take the root mean square of the base vector values.

where for our purposes $p_i = u_i / \Sigma_i u_i$.[7] This measure of entropy reaches its maximum of one when development is uniformly distributed across the study area (so that $p_i = 1/n$, $\forall i \in N$), and it is minimized at zero when all development is concentrated in one location j (so that $p_j = 1$, and $p_i = 0$, $\forall\ i \neq j$). Thus, in the context of figure 5, the central [$\varnothing$ - X] diagonal represents perfect entropy, and entropy decreases with the angle away from this central diagonal. A word of caution: Kosko (1992) refers to the midpoint M in his fuzzy power set as the point of maximum entropy, and argues that entropy is a measure of the fuzziness of a set. We have chosen not to retain that terminology because it blurs the distinction between equations 7 and 8 above. While it is true that the midpoint M corresponds to perfect entropy under either definition, the same does not hold for other points along the central diagonal.[8]

Taken as a package, these three dichotomies provide a highly comprehensive characterization of the status and scope of urbanization for a study area. Not only does this approach provide a measure of the extent of urbanization, it also characterizes an urbanizing system in terms of its degree of fuzziness and its level of entropy.

Application to Ningbo, China

We contend that the three measures operationalized in equations 6, 7, and 8 provide a comprehensive set of urbanization statistics well suited to measuring, monitoring, characterizing, and comparing fuzzy urban systems. As remote sensing data become increasingly available urbanization researchers will become less dependent upon jurisdiction-bound statistics that are ill-suited to conveying the contours of *desakota* phenomenon, and so we envision significant scope for widespread adaptation of this approach. To test the feasibility of our proposed method and approach we turn now to a pilot application. The study area we have selected is Ningbo City, Zhejiang Province, in eastern China south of Shanghai. Ningbo is an active port area and an industrial center on the Yong River. Located in a fertile agricultural region, it is well known for its rural industry.[9] This case study was selected in part because of the rapid and diffuse patterns of urbanization found there and, in part, because of data availability. Our test case uses *Landsat* TM2, TM5, and TM7 bands for

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[7] Yeh uses density of development as the underlying variable. From our perspective, development density is a reasonable basis for measuring membership in U, and so the two approaches are basically consistent.

[8] Consider, for example, the case where all parcels are twenty percent urban, so that $u_i = 0.20$ $\forall i \in N$. Using the conventional definition of entropy as expressed in equation 8, this condition corresponds to perfect entropy, or E = 1. However, using Kosko's definition from equation 7, this condition corresponds to F = 0.20/0.80 = 0.25.

[9] See http://www.ningboport.com/

three points in time; 1987, 1990, and 1996; each with 30m x 30m resolution and a 350 x 312 pixel image.

Recall from our discussion of figure 2, that the starting point for our method is the remote sensing image. There are two basic steps in moving from [2 → 3] in the context of figure 2. The first step converts the remote sensing image to a representation of the fuzzy set *U*, while the second set derives the three summary statistics described above. Figure 6 illustrates this first step. Remote sensing data are generated by measuring the percentage reflection across differing wave frequencies. Each type of land cover material has its own unique spectral signature indicating the reflective pattern across the wave spectrum. Remote sensing data sample discrete wavelength bands, and so each pixel in our dataset has a (percentage reflection) datum associated with each of the three bands. The first task, therefore, is to convert these three data into a membership value u_i for each pixel *i*. Jensen (1996) provides a useful overview of multispectral supervised classification methods that are commonly used for this purpose. A more detailed discussion of fuzzy classification methods of remote sensing data is provided by Wang (1990). For our application we use a modified distance-to-means technique that has simplicity as its chief virtue. Several samples of pixels from the image are selected manually (with the aid of Visual C++ programming tools) as being most representative of core urbanized areas. The average reflective value for these training sites is computed for the sample, and the distance from this sample mean value is then calculated for each pixel. The degree of membership in *U* for each pixel *i* is then formulated as the normalized complement of this distance measure. This enables us to produce a remote sensing image for each year whose shades of gray represent degrees of membership in the fuzzy set U, as indicated in our discussion of figure 3. It also provides the basic input data needed to perform the calculations specified in equations 6, 7, and 8.

figure 6 here: "Converting image data to fuzzy set data"

The results of this test case are summarized in table 2.

Table 2: Ningbo City as a fuzzy urban system

Year	***Level of urbanization***	***Extent of fuzziness***	***Degree of Entropy***
1987	0.519	0.475	0.993
1990	0.518	0.471	0.993
1996	0.530	0.460	0.993

The experiment was repeated two additional times, and those results are reported in the appendix. The results from each trial are quite similar qualitatively and quantitatively, and this suggests that the method is reasonably robust. In each case, the results point to the following conclusions:

- The measured level of urbanization in the study area remained essentially unchanged from 1987 to 1990, but increased perceptibly from 1990 to 1996. In the context of figure 5 this implies a movement away from the origin ∅.
- The fuzziness of the urbanizing system decreased steadily from one period to the next. This tells us that land use patterns are beginning to articulate themselves more distinctively, with clearer demarcations between urban and rural land uses. In the context of figure 5, it denotes a steady movement away from the midpoint M and towards one of the outer vertices.
- The level of entropy within the system is quite high, indeed it is almost complete. Moreover, there is no perceptible change in entropy from one period to the next. In the context of figure 5 this tells us that the urban set U is located somewhere along the central diagonal.

Taken together, these results tell us that the representation of the Ningbo City study area in the context of Kosko's fuzzy hypercube is along the central diagonal, moving slowly away from *M* and towards *X*, the point of complete urban saturation. It tells us that the entire region is urbanizing in a highly diffused manner, but that the process is not yet complete and so a relatively high level of urban ambiguity remains.

Concluding Remarks

From the perspective of the Chinese government, continued urbanization is a top priority for China's economic development and "a symbol of continuous human progress and the historical trend of the contemporary world".[10] At present, just over thirty percent of China's population is regarded as urban, compared to less than twenty percent in 1978. With a total population of 1.2 billion and growing, the implications of aggressive urbanization in China are staggering. The State Development Planning Commission, the Ministry of Construction, the World Bank, and many other national and international agencies are struggling to help formulate policies to facilitate and promote effective urbanization[11]. A central question in this regard concerns the appropriate spatial form and distribution of urbanization. Although

[10] See remarks by Zhao Baojiang, Vice Minister of China's Ministry of Construction at World Bank (2000) workshop on urbanization in China.

[11] See Chan (1992) or Ma and Fan (1994) for a contemporary historical perspective.

urban geographers are well aware of the prevalence and significance of *desakota* urbanization patterns in China and elsewhere, urban economists have been much less inclined to focus on this phenomenon. One reason for this reluctance is that spatially complex diffusion patterns cannot be summarized or characterized succinctly, and therefore are not readily incorporated into the kinds of formal models that economists are accustomed to using. As we have seen, urban geographers are moving more quickly from descriptive narratives of *desakota* phenomena to a range of computer-based modeling approaches. However, even in the absence of formal models, summary data are important for our ability to grasp the evolving nature and extent of urbanization in China and elsewhere.

The method proposed in this paper addresses this challenge in a unique way. The approach is at once simple yet comprehensive. By formulating an urbanizing region (or a remote sensing image thereof) as a fuzzy urban set, we can characterize its development in terms of three fundamental dichotomies: (i) urban-rural, (ii) fuzzy-crisp, and (iii) entropy-order. Moreover, these three measures each stem from a single geometric interpretation whereby the fuzzy urban set U is located as a single point within Kosko's fuzzy cube. A comprehensive characterization of the observed urbanization phenomenon is derived simply by describing the fuzzy set U within this unifying context. This provides a solid basis for comparing urbanization patterns over time and in different places, and it provides succinct measures of urbanization that may in turn support the development of new classes of urbanization models.

Having said this, many issues remain to be addressed before this approach can be adapted widely and systematically. Foremost among these is the question of scope and scale of the areas to be compared. The measured attributes of any given study area will vary with one's choice of the corresponding remote sensing image boundary or resolution[12], and it is difficult to avoid a certain element of arbitrariness in this selection. Regarding boundary issues, one practical approach is to work within existing jurisdictional boundaries and measure urbanization trends within this framework. An advantage of this approach is that it provides a frame of reference which, albeit arbitrary, has meaningful links to decision-making. This is less useful, however, wherever the focus of urbanization extends across jurisdictional boundaries, as is increasingly the case. Another disadvantage of using jurisdictional boundaries is the pitfall of circular reasoning alluded to earlier whereby urban places are defined, in effect, as *exogenously defined places* that meet some criteria for "urban".

[12] For example, the near-perfect entropy found in our test case may be attributed to remote sensing image boundaries that fit snugly to the urbanizing region. A more "zoomed out" image would likely reduce the observed entropy as the urbanization phenomenon would be more tightly concentrated in a distinct subregion of the image.

Working with fuzzy urban sets softens this dilema somewhat by allowing degrees of urbanity, but it does not address the issue of place definition in a more fundamental geographic sense.[13] Even if place (or remote sensing image) boundaries were agreed upon, one's measure of urbanization is likely to depend upon the resolution of the image itself. Of course, these issues are not new, nor are they specific to the fuzzy urban set approach advocated here, but they are issues that must be contended with before our proposed method can be subject to systematic and widespread use. Notwithstanding these challenges, we aver that *fuzzy urban sets* are a useful paradigm for characterizing urbanization trends in China and other regions where *desakota* patterns are the norm.

Appendix

The complete results from three separate trials are presented here. The first set also appears in table 2 of the main text. The trials differ in terms of the training areas selected.

First Trial

Year	*Level of urbanization*	*Extent of fuzziness*	*Degree of Entropy*
1987	0.519	0.475	0.993
1990	0.518	0.471	0.993
1996	0.530	0.460	0.993

Second Trial

Year	*Level of urbanization*	*Extent of fuzziness*	*Degree of Entropy*
1987	0.561	0.488	0.994
1990	0.560	0.490	0.994
1996	0.564	0.485	0.994

Third Trial

Year	*Level of urbanization*	*Extent of fuzziness*	*Degree of Entropy*
1987	0.548	0.444	0.992

[13] A more promising approach may be to define an urban place more endogenously in cartographic terms as an aggregation of urban pixels, although this too invites many more questions regarding spatial autocorrelation, contiguity, etc.

1990	0.547	0.443	0.992
1996	0.559	0.428	0.992

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Figure 1: McGee's

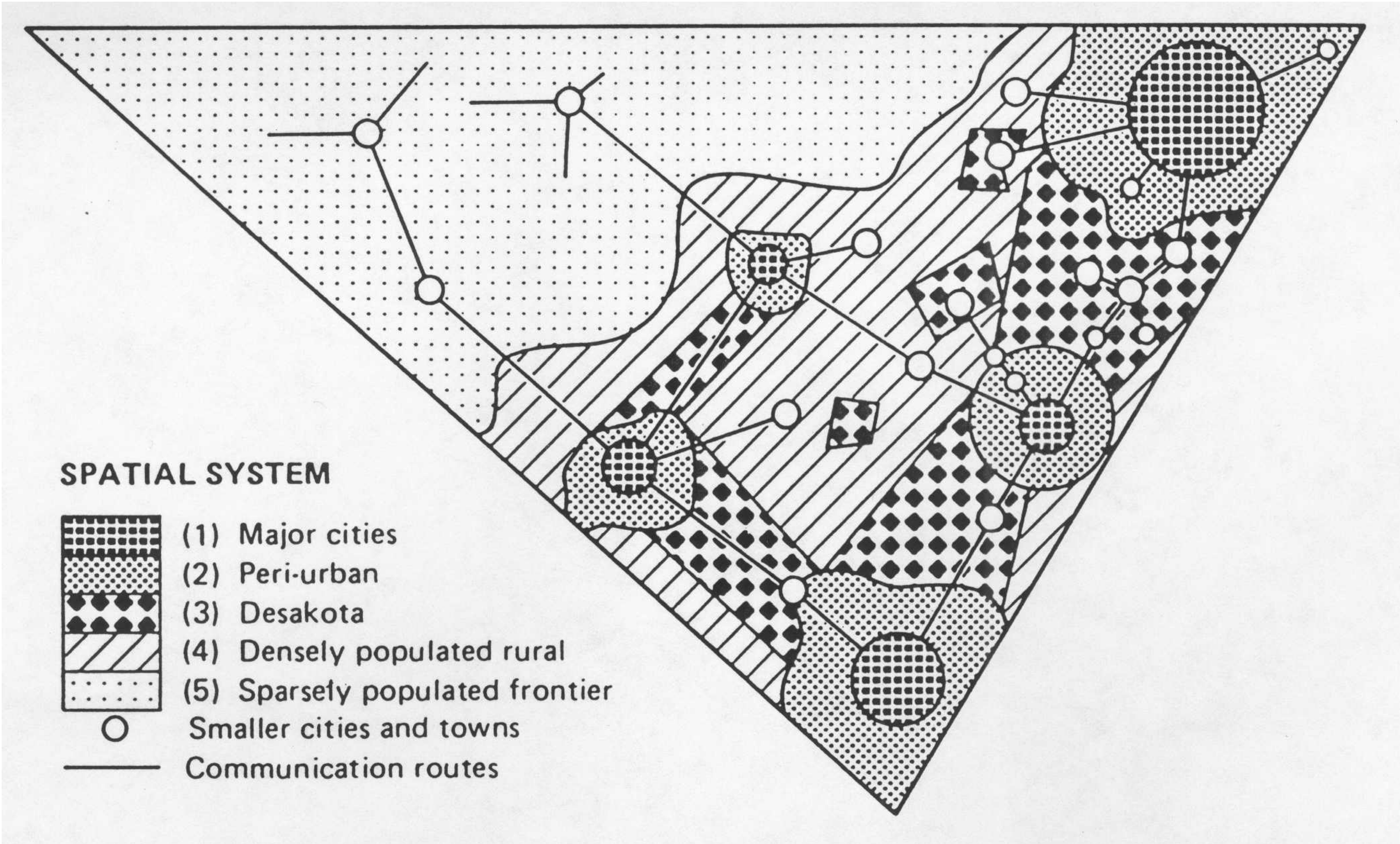
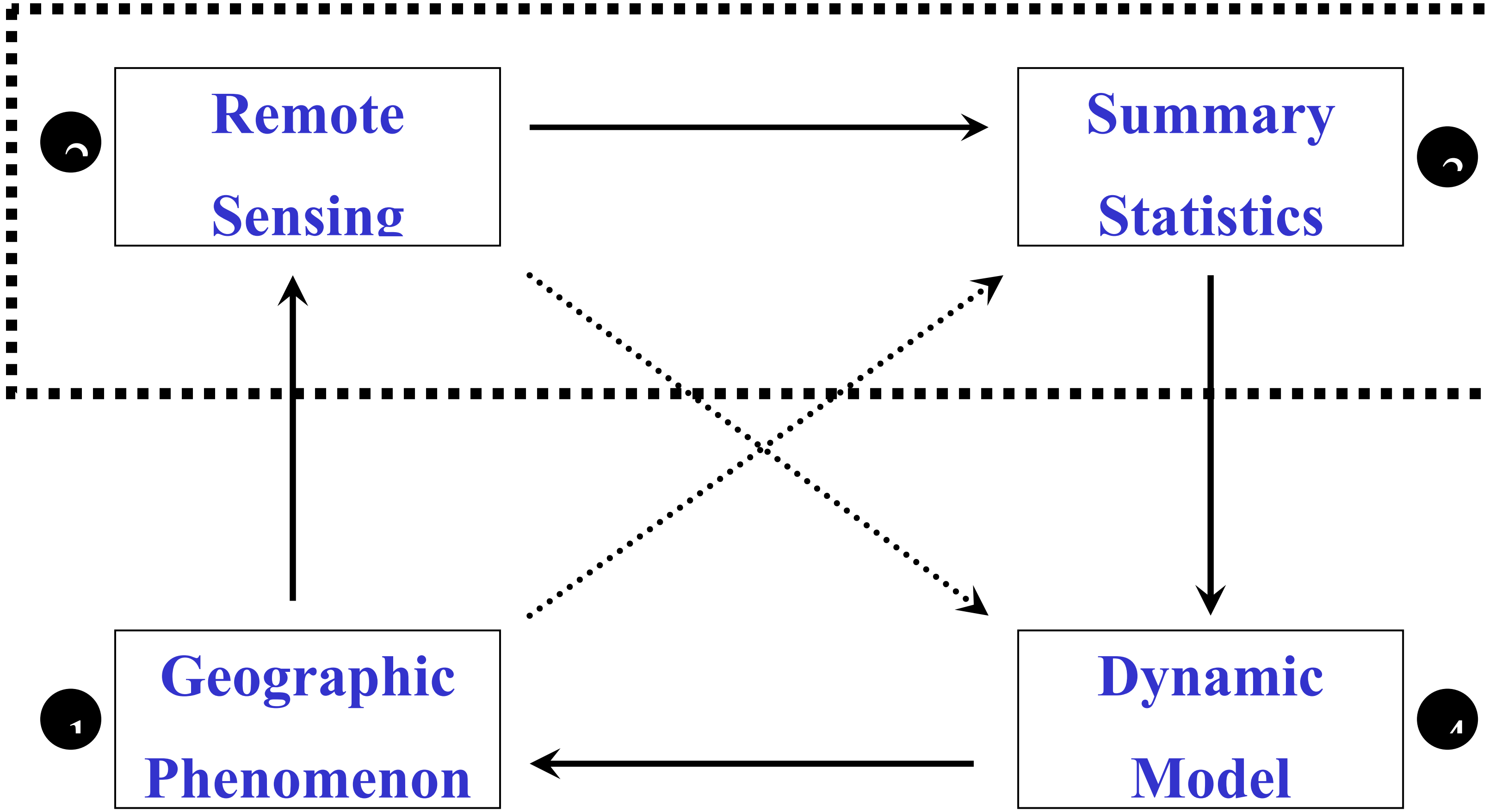


Figure 2: Context of



The primary focus of this paper is on [2 →3]

Figure 3: Remote sensing image of Ningbo

Figure 4: One dimensional urban sets (fuzzy and

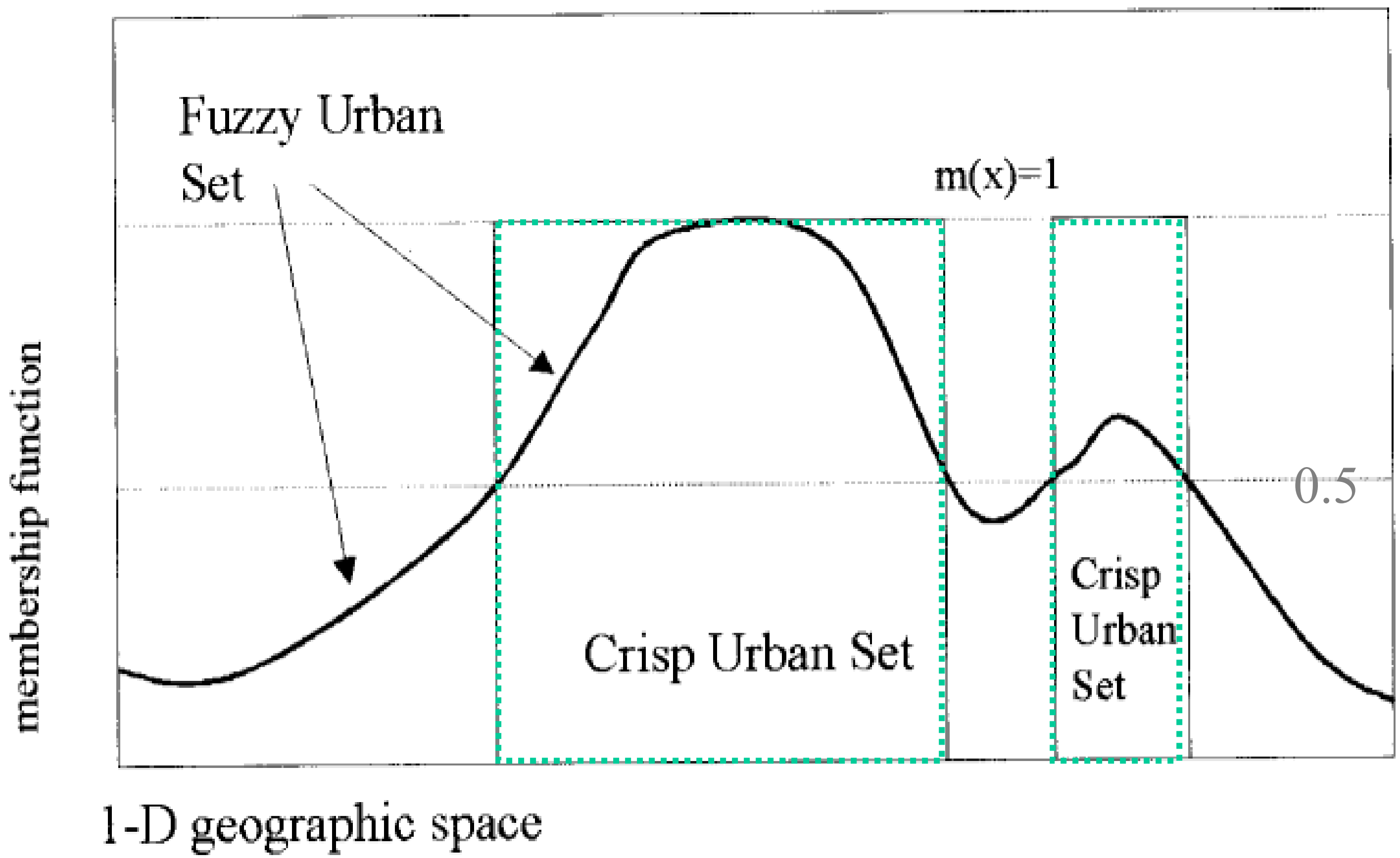


Figure 5: Kosko's fuzzy hypercube

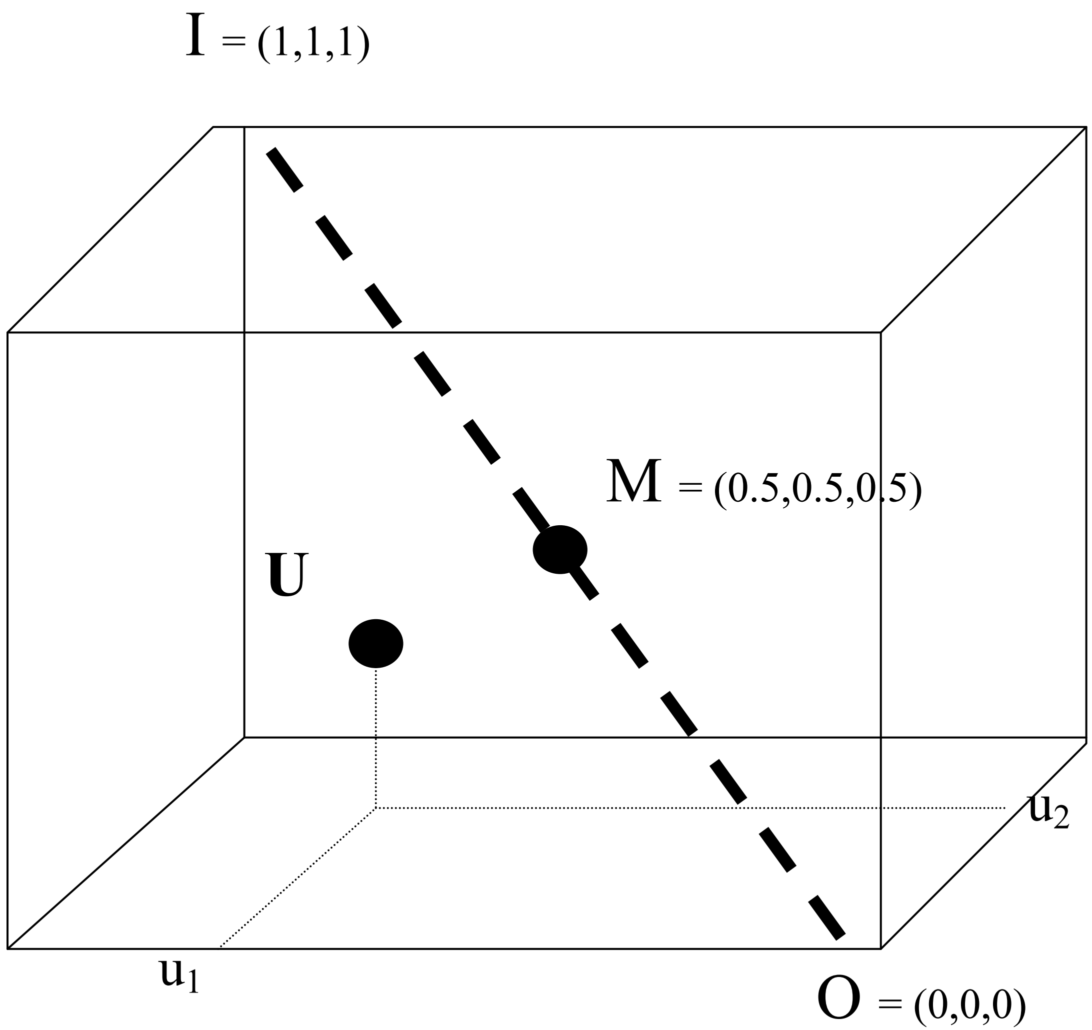


Figure 6: Converting image data to fuzzy set