Investigation on spatial relations in 3D GIS based on NIV

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Abstract In this paper the shortages of 4-intersection model and 9-intersection model for describing spatial relations are discussed. Furthermore, an improved model for describing spatial relations is proposed, called NIV, and the characters of NIV are introduced. And then, NIV model is extended to the three dimensional space, including the definitions of 3D spatial entities, generation of 3D Voronoi diagram and line for describing 3D spatial relations based on 3D Voronoi diagram. It shows that NIV is feasible for description of 3D spatial relations.

1. Background

Although our perception and actual universe is three-dimensional, spatial features in geographical information system are for the time being mostly managed in two-dimensional ways. Therefore, most of the present activities concentrate on the three-dimensional GIS. In the field of 3D GIS research, the spatial relation is the basis of 3D data structure and 3D data model. So many scientists pay attentions to this field. In 1999, Corbert explained the topological model in 2D GIS. In 1993, 9-intersection model was used for investigating the spatial relations in 3D GIS. Guo (1998) induced the description of spatial relations in 3D space between simple object such as point, line and area results via 9-intersection model. As testified by Li in 1997, the 9-intersection model is not perfect in theory, even is not better than 4-intersection model. The reasons can be found in Li's papers, and as 9-intersection model's improvement, the NIV model was also proposed.

2 Development of NIV and its extension to 3D space

2.1 Existed framework of describing spatial relations

A formalization of topological relations has been investigated in later 1980s based on point-set topology [Guting, 1988; Pullar, 1988; Egenhofer and Franzosa, 1991]. The result of such a formalization is the so-called 4-intersection, which can describe completely topological relations between simple spatial objects (i.e. simple point, line and area). Its basic principle is to regard any of spatial objects as the set consisted of its two topological components, i.e., boundary and interior, then the intersections of two components will consist of a 2×2 matrix，noted as

$$R_4(A, B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^0 \\ A^0 \cap \partial B & A^0 \cap B^0 \end{bmatrix}$$ （1）

Here, ∂A、∂B denotes the boundary set of spatial objects *A* and *B* respectively; A^0、B^0 denoting the interiors. According to the formula (1), two kinds of topological relations for point/point, 3 for point/line, 3 for point/area, 16 for line/line, 13 for line/area and 8 for area/area can be distinguished. But there are still some topological relations between line and line, and line and area, which can not be distinguished via the 4-intersection model, that is to say, some different topological relations has the same description matrix. For this reason, Egenhofer et al（1991）made n extension to the 4-intersection model, leading to a new model called the 9-intersection model , as follows:

$$R_9(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^0 & \partial A \cap B^- \\ A^0 \cap \partial B & A^0 \cap B^0 & A^0 \cap B^- \\ A^- \cap \partial B & A^- \cap B^0 & A^- \cap B^- \end{bmatrix} \qquad (2)$$

Here, A^0, ∂A and A^- mean the interior, boundary and exterior of A, respectively. The annotation for B is the same. In this model, the exterior of A is normally defined as the complement. It is clear that some of the limitations associated with the 4-intersection model are overcome in this model. And two kinds of topological relations for point/point, 3 for point/line, 3 for point/area, 23 for line/line, 19 for line/area and 8 for area/area can be distinguished based on 9-intersection model. It is clear to find that additional seven kinds of topological relations for line/line and six kinds of topological relations for line/area are distinguished under 9-intersection model, which can't be distinguished under 4-intersection model. For other spatial relations such as point/point, point/area, area/area etc, 9-intersection model is equivalent with 4-intersection model. But such topological relations can't be distinguished via 9-intersection model as the intersection set is point, line and area set respectively. For this reason, a so-called dimension extended method is put forward by Clementini *et al* (1993), i.e., to use 0D, 1D and 2D to identify when the intersection set is point, line and area set respectively, if required.

2.2 Imperfection of the 9-Intersection model

From above discussions, it is easy to know that more topological relations can be described based on 9-intersection model than 4-intersection model, especial topological relations for line/line and line/area. This is because introduction of the complement of spatial object will solve co-dimension property of line object. But some other problems on description of topological relations are still open, as follows:

(1) Too large spatial ranges of the exterior

In the 9-intersection model, suppose that A is one spatial object, then the exterior of spatial object A, i.e. A^-, is defined as its complement set in point-set topological space, as leads to the fact that such a relation is always satisfied, that is, $A^- \cap B^- = -\phi$. However, this will weaken the functionality of 9-intersection model for description of topological relations.

(2) Linear correlation of three components of spatial entity

From the formula (2) we may find that, the three components of spatial object set, i.e., its boundary, interior and exterior, satisfying:

$$\begin{cases} C = A^0 + \partial A + A^- \\ C = B^0 + \partial B + B^- \end{cases} \qquad (3)$$

Here, the annotation for A^0, ∂A, A^-, B^0, ∂B and B^- is the same as above; C denotes the total point-set topological space. For formula (3), it can be transformed as follows:

$$\begin{cases} A^- = C - (A^0 + \partial A) \\ B^- = C - (B^0 + \partial B) \end{cases} \qquad (4)$$

By substituting (4) into (2), the following model is obtained:

$$R_9(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^0 & \partial A \cap (C - B^0 - B^-) \\ A^0 \cap \partial B & A^0 \cap B^0 & A^0 \cap (C - B^0 - B^-) \\ (C - A^0 - A^-) \cap \partial B & (C - A^0 - A^-) \cap B^0 & A^- \cap (C - B^0 - B^-) \end{bmatrix} \qquad (5)$$

Obviously, it means that, given an object A, its complement is completely determined by a linear function of C and itself. As C is a constant space, therefore, the exterior (defined as the complement), boundary and interior

of *A*(or *B*) are linearly dependent. Therefore, there is a one-dimensional redundancy in this model. Theoretically, such definition of the complement will lead to the fact that some topological relations can't be distinguished under this model.

(3) Confusion of spatial disjoint relations

Suppose that there is another entity in between *A* and *B*, if based on the 9-intersection model, spatial relationship between spatial entity *A* and *B* is adjacent relations, while in fact spatial relationship is an isolate relation, because there is another entity in between *A* and *B*, which leads such a fact that spatial entity *A* must cross another entity to access *B*. Obviously, these two spatial relations are different, but the results are the same based on above 9-intersection model.

(4) Only feasible for simple spatial entities

The 9-intersection model is only feasible to describe spatial relations between simple objects as the homogeneously 2-dimensional, connected areas and lines with exactly two end points [Egenhofer, 1993], and it is difficult to deal with complex objects such region with holes. For this reason, Egenhofer et al (1994) still use the 9-intersection model, but they will regard the region with holes as several separate objects, and use induction method to determine spatial relations. Obviously, this model is more difficult and complex to consider spatial relations between complex objects.

Based on above discussion, a new Voronoi-based 9-intersection (NIV) was proposed in the 2.3 section. With the new VNI, it is possible to distinguish disjoint relations, to deal with objects with holes and to compute the five exterior-based intersections.

2.3 What is NIV

In 1997, Li Chengming etc. proposed the NIV model, in this model the voronoi region of object replaces the complement of object. Each object can be divided into three components, i.e., boundary, interior and voronoi region. The intersection of object A's three components and object B's three components consist 9-intersection based on voronoi region model (NIV). In formula 6, ∂A denoted boundary of object, A° denoted the interior of object A, A^V denoted the exterior of object A, is the voronoi region of object A.

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^\circ & \partial A \cap B^V \\ A^\circ \cap \partial B & A^\circ \cap B^\circ & A^\circ \cap B^V \\ A^V \cap \partial B & A^V \cap B^\circ & A^V \cap B^V \end{bmatrix} \tag{6}$$

3. Spatial relations in 3D space based on NIV

3.1 Definition of spatial entities in 3D GIS

In 2D space, point, line and area were considered as the basic geographical model, the other spatial objects can be represented by the combination of basic entities, thus the spatial relation between objects can be changed into spatial relations of basic geographical models. In 3D space, basic spatial entities will be extended as point, line, surface and body, and the four spatial entities or the combination can represent all spatial objects. Similarly, their geometric shape, position and topological relations will be so-called spatial properties. Based on the point-set topology, 3D spatial entities can be defined as follows:

(1) Point entity is 0-dimensional, which can be defined by its coordinate (x, y, z). A point entity will be mapped as one 0-simplex with its location information, but it only can represent one spatial position without any other spatial properties such as length, area etc. In reality, it can denote such entities as well, telegraph pole, tree etc.

(2) Line entity is 1-dimensional, denoting a continuous curve or straight line. It only has the length property, thus it may be used to represent road, river and communication circuitry etc in 3D space. For one line entity, it may be closed, or has several branches, therefore, it consists of some connected 1-simplex with one direction.

(3) Surface entity is 2-dimensional. It has geometric length and area properties. For one surface entity, it can be partitioned finite 2-simplex.

(4) Body entity is 3-dimensional spatial object, which can be measured by its volume and surface area. And any of body entity can be partitioned as finite connected 3-simplex.

3.2 Voronoi diagram formation of 3D spatial entities

The Voronoi diagram is also known as a Thiessen diagram, Wigner Seitz cells or Dirichlet tessellation. Its basic purpose is used for the description of boundaries of the so-called 'region of influence' or 'spatial proximity' for each of a set of spatial data points [Brassel and Reif, 1979]. In the past, there are two fundamental approaches for Voronoi diagram computation. One is to build a triangular network first and then to derive the Voronoi diagram from the triangular network if the relationship between Voronoi diagram and a triangular network is unique, another to generate the Voronoi diagram directly from spatial data points and to derive the triangular network from the Voronoi diagram, if required. And these two approaches are based on vector mode and are good for point sets. For line and area sets, it seems that no efficient rigorous algorithms have been developed. In order to this, C.Li et al. proposed a new method. Its basic idea is to compute Voronoi diagram by means of dynamic distance transformation, some detailed realization steps could be found in Li's original paper [Li, et al, 1999]. It is noted that, the method is based on raster mode and feasible for spatial point, line and area sets, and the Voronoi diagram for line and area sets can be computed as easily as for point sets. In the meanwhile, Li et al. (1999) also pointed out that the raster-based methods could easily be extended to the 3D Voronoi diagram. Through the efforts of authors, Voronoi diagram computation of 3D spatial entities including spatial geometric body have been realized. Indeed, The basic idea of Voronoi diagram generation of 3D spatial entities is the same as the one in 2D space. A simple example of Voronoi diagram computation for two 3D spatial points are illustrated in Fig.1.
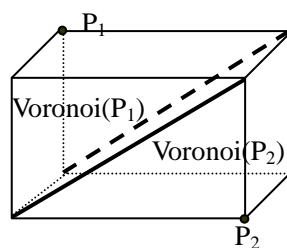

Fig1. 3D Voronoi Diagram of two points P_1 and P_2

3.3 Description of 3D spatial relations based on Voronoi diagram

The discussions presented in §3.1 has shown that there are four basic features, that is, spatial point, line , area and body in 3D space, the other spatial objects can be represented by the four features or their combination. Therefore, the spatial relation between spatial objects can be changed into spatial relations of basic geographical models. According to the above definitions, their topological relations can be classified as ten, i.e., point/point, point/line, point/area, point/body, line/line, line/area, line/body, area/area, area/body and body/body. For each of these topological relations, it may still be described via NIV like formula (2), but their elements denote the intersection of the components of two 3D spatial objects.

4. Conclusions

In this paper, three conclusions can be drawn, one is that the formal definitions of basic models in 3D space were given, the second is to prove the NIV model doable when extending to 3D space from 2D space, at last the description results and processes are given in this paper.

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